NTPL
ALL INDIA TEST SERIES

CODE - A

TEST ID 001922

JEE (Main) - 2019

FULL TEST - 7

Time : 3 Hours | *Maximum Marks : 360*

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

You are not allowed to leave the Examination Hall before the end of the test.

INSTRUCTIONS

A. General :

1. This booklet is your Question Paper containing **90 questions**.
2. The Question Paper **CODE & TEST ID** is printed on the right hand top corner of this booklet. This should be entered on the OMR Sheet.
3. Fill the bubbles completely and properly using a **Blue/Black Ball Point Pen** only.
4. No additional sheets will be provided for rough work.
5. Blank papers, clipboards, log tables, slide rules, calculators, cellular phones, pagers, and electronic gadgets in any form are not allowed to be carried inside the examination hall.
6. The answer sheet, a machine-readable Optical mark recognition sheet (OMR Sheet), is provided separately.
7. DO NOT TAMPER WITH / MUTILATE THE OMR OR THE BOOKLET.
8. Do not break the seals of the question-paper booklet before being instructed to do so by the invigilator.

B. Question paper format & Marking Scheme :

9. The question paper consists of **3 parts** (Physics, Chemistry and Maths).
10. The test is of **3 hours** duration. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct. Each question carries **+4 marks** for correct answer and **-1 mark** for wrong answer.

Name of the Candidate (in Capitals) ____________________

Test Centre ____________________ Centre Code ____________________

Candidate's Signature ____________________ Invigilator's Signature ____________________

PHYSICS

SECTION – I

This section contains ***30 questions.*** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which ***ONLY ONE*** is correct. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

1. A swimmer can swim in still water with a speed of $\sqrt{5}$ m/s. While crossing a river his average speed is 3 m/s. If he cross the river in the shortest possible time, what is the speed of flow of water?

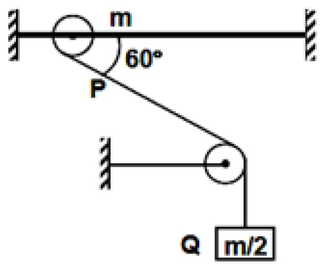
(a) 2 m/s
(b) 4 m/s
(c) 6 m/s
(d) 8 m/s

2. A car starting from rest is accelerated at constant rate until it attains a constant speed v. It is then retarded at a constant rate until it comes to rest. Considering that the car moves with constant speed for half of the time of total journey, the average speed of the car for the journey is

(a) $\frac{v}{4}$
(b) $\frac{3v}{4}$
(c) $\frac{3v}{2}$
(d) Data insufficient

3. A smooth ring P of mass m can slide on a fixed horizontal rod. A string tied to the ring passes over a fixed pulley and carries a block Q of mass (m/2) as shown in the figure. At an instant, the string between the ring and the pulley makes an angle 60° with the rod.

The initial acceleration of the ring is

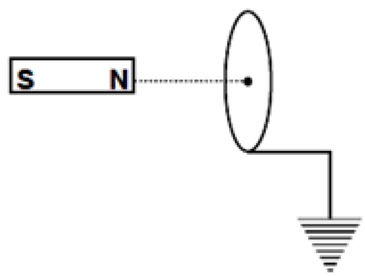


(a) $\frac{2g}{3}$
(b) $\frac{2g}{6}$
(c) $\frac{2g}{9}$
(d) $\frac{g}{3}$

4. A block of mass M is hanging over a smooth and light pulley through a light string. The other end of the string is pulled by a constant force F. If kinetic energy of the block increases by 20 J in 1s. Then

(a) tension in the string is Mg.
(b) tension in the string is F
(c) Work done by the tension on the block is 20 J in 1 sec.
(d) Work done by the force of gravity is 20 J in 1 sec.

5. A circular ring is fixed in a gravity free space and one point of the ring is earthed. Now a magnet is placed along axis of the ring at a distance from its centre such that the nearer pole is north pole as shown in figure. A sharp impulse is applied on the magnet so that it starts to move towards the ring. Then,



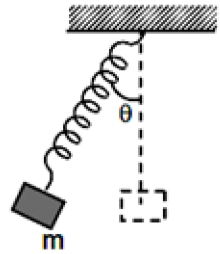
(a) Initially magnet experiences an acceleration and then it retards to come to an instantaneous rest.

(b) Magnet starts to oscillate about centre of the ring

(c) Magnet continuous to move along the axis with constant velocity

(d) The magnet retards and comes to rest finally.

6. A block of mass m is suspended by means of an ideal spring of force constant k from ceiling of a car which is moving along a circular path of radius 'r' with acceleration 'a'. The time period of oscillation of the block when it is displaced along the spring, will be



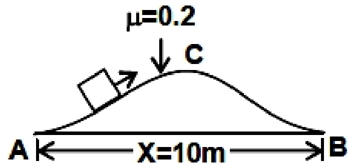
(a) $2\pi\sqrt{\frac{mg+ma}{k}}$ (b) $2\pi\sqrt{\frac{m}{k\sqrt{g^2+a^2}}}$

(c) $2\pi\sqrt{\frac{m}{k}}$ (d) $2\pi\sqrt{\frac{m}{\sqrt{k^2+g^2+a^2}}}$

7. In the hydrogen atom spectrum λ_{3-1} and λ_{2-1} represent wavelengths emitted due to transition from second and first excited states to the ground state respectively. The value of $\frac{\lambda_{3-1}}{\lambda_{2-1}}$ is

(a) 27 / 32 (b) 32/27 (c) 4/9 (d) 9/4

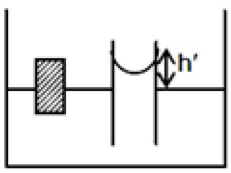
8. A block of mass 1 kg is pulled along the curve path ACB by a tangential force as shown in figure. The work done by the frictional force when the block moves from A to B is



(a) 5 J (b) 10 J

(c) 20 J (d) none of these

9. In a capillary tube placed inside the liquid of density (ρ) in a container, the rise of liquid is h. When block of density 'σ' is placed on the liquid as shown in figure, liquid in the tube is h'. If $\sigma < \rho$ then



(a) $h' = h$

(b) $h' < h$

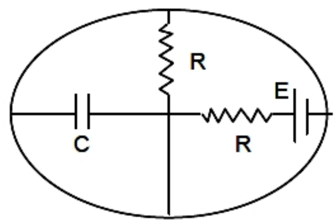
(c) $h' > h$

(d) insufficient data

10. A light rod of length L, is hanging from the vertical smooth wall of a vehicle moving with acceleration $\sqrt{3}$ g having a small mass attached at it's one end is free to rotate about an axis passing through the other end. The minimum velocity given to the mass at it's equilibrium position so that it can complete vertical circular motion is

(a) $\sqrt{5gL}$ (b) $\sqrt{4gL}$ (c) $\sqrt{8gL}$ (d) none of these

11. Energy stored in the capacitor in it's steady state is



(a) CV (b) $\frac{CV}{2}$

(c) $\frac{1}{2}QV$ (d) zero

12. The potential difference between two points A and B which are separated by a distance of 1m along the field in a uniform electric field of $10NC^{-1}$ is

(a) zero (b) 100 volt (c) 10 volt (d) 0.1 volt

13. The power factor of a circuit in which a box having unknown electrical devices connected in series with a resistor of resistance 3Ω is 3/5. The reactance of the box is

(a) $5\,\Omega$ (b) $5/3\,\Omega$ (c) $4\,\Omega$ (d) $4/3\,\Omega$

14. Two points A and B are at distances of 'a' and 'b' respectively from an infinite conducting plate having charge density σ. The work done in moving charge Q from A to B is

(a) $\frac{Q\sigma}{\varepsilon_0}(b-a)$ (b) $\frac{\sigma}{(b-a)}Q$ (c) $\frac{Q\sigma}{(b-a)\varepsilon_0}$ (d) none of these

15. A point charge of 0.1C is placed on the circumference of a non-conducting ring of radius 1m which is rotating with a constant angular acceleration of 1 rad/sec^2. If ring starts it's motion at t = 0 the magnetic field at the centre of the ring at t = 10 sec, is

(a) 10^{-6} T (b) 10^{-7} T (c) 10^{-8} T (d) 10^{7} T

16. The wavelength corresponding to maximum spectral radiancy of a black body A is λ_A = 5000 Å. Consider another black body B whose surface area is twice of that of A and total radiant energy emitted by B is 16 times that emitted by A. The wavelength corresponding to maximum spectrum radiancy for B will be

(a) $5000\,(8)^{1/4}$ Å (b) 2500 Å (c) 10, 000 Å (d) $\frac{5000}{(8)^{1/4}}$ Å

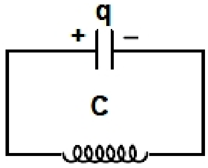
17. During an adiabatic process, the density of a gas is found to be proportional to cube of temperature. The degree of freedom of gas molecule is

(a) 6 (b) 5 (c) 4 (d) 3

18. Two fixed charges $-2Q$ and Q are located at the point of co-ordinates $(-3a, 0)$ and $(3a, 0)$ respectively in $(x-y)$ plane. Then all the points in x-y plane where potential is zero lies on a

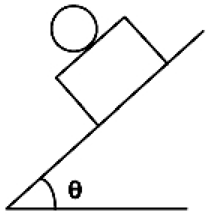
(a) straight line parallel to x-axis
(b) straight line parallel to y-axis
(c) a circle of radius $4a$
(d) circle of radius $2a$

19. In an L-C circuit shown in the figure, C = 1F, L = 4H. At time t = 0, charge in the capacitor is $4C$ and it is decreasing at a rate of $\sqrt{5}$ C/s. Choose the correct statements.



(a) maximum charge in the capacitor can be 6C
(b) maximum charge in the capacitor can be 8C
(c) charge in the capacitor will be maximum after time $2\sin^{-1}(2/3)$ sec
(d) None of these

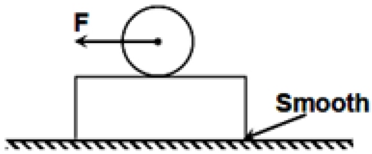
20. A plank of mass M is placed over smooth inclined plane and a sphere is also placed over the plank. Friction is sufficient between the sphere and the plank to prevent slipping. If the plank and the sphere are released from rest the frictional force on the sphere is



(a) up the plane
(b) down the plane
(c) zero
(d) horizontal

21. A disc of mass m and radius R is placed over a plank of same mass m. There is sufficient friction between the discs and the plank to prevent slipping. A force F is applied at the centre of the disc.

Choose the correct statements.

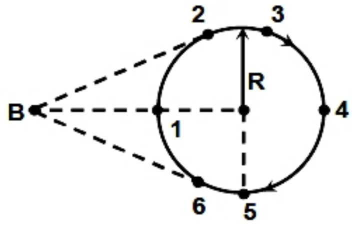


(a) Acceleration of the plank is $\frac{F}{4m}$
(b) Acceleration of the plank is $\frac{F}{2m}$
(c) Force of friction between disc and plank is $\frac{F}{6}$
(d) Force of friction between disc and plank is $\frac{F}{2}$

22. A certain amount of ideal monoatomic gas undergoes, process given by $UV^{1/2} = C$ where U is the internal energy of the gas. The molar specific heat of the gas for the process will be

(a) R/2
(b) 3R
(c) 5R/2
(d) –R/2

23. An isotropic sound source A is moving in a circle of radius R with a small speed v. An observer B is hearing this sound (See figure). The intensity of the sound heard by B will be maximum when the source is at point.



(a) 1

(b) 2

(c) 6

(d) none of these

24. A stable species C can be obtained through unstable nuclides A and B. The half life period for conversion A $\rightarrow$ C is T and that for conversion B $\rightarrow$ C is 2T. Initially a sample contains N nuclides of A, N nuclides of B and no nuclides of C. After how much time will the nuclides of species C be equal to (27/16) N

(a) 2T (b) 3T (c) 4T (d) 5T

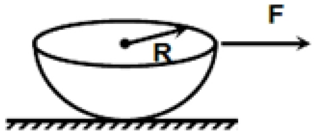
25. A certain thermodynamic cycle comprises of two isothermal and two adiabatic processes. The highest temperature obtained in the entire cycle is 600 K while the lowest temperature is 300 K. When the cycle represented on a P-V curve the sense of the cycle is counter clockwise. The efficiency of the cycle is

(a) 50% (b) 75 % (c) 25 % (d) None of these.

26. A man wearing spectacles with diverging lenses is viewing a distance object. The image formed on his retina will be

(a) real and erect (b) real and inverted (c) virtual and erect (d) virtual and inverted

27. A thin uniform hemispherical bowl of mass m and radius R is lying on a smooth horizontal surface. A horizontal force F is now applied perpendicular to the rim of the bowl (see figure). The instantaneous angular acceleration of the bowl will be



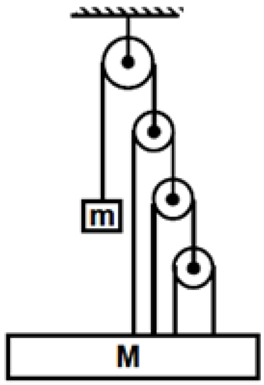
(a) $\frac{20}{3}\frac{F}{MR}$ (b) $\frac{10}{3}\frac{F}{MR}$

(c) $\frac{40}{3}\frac{F}{MR}$ (d) none of these

28. For a stone thrown from a tower of unknown height, the maximum range for a projection speed of 10 m/s is obtained for a projection angle of 30°. The corresponding distance between the foot of the tower and the point of landing of the stone is

(a) 10 m (b) 20 m (c) $\left(20/\sqrt{3}\right)$m (d) $10\sqrt{3}$m

29. Consider the system shown in the figure. the system is so arranged that both m and M are in pure translation. If at a certain instant M is moving down with a speed of 1 m/s then the speed of m will be



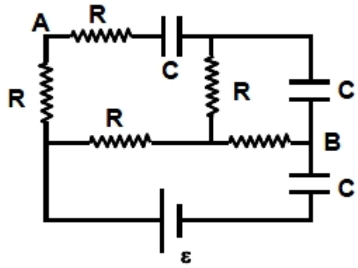
(a) 7 m/s

(b) 8 m/s

(c) 1 m/s

(d) 15 m/s

30. Consider the circuit in the adjacent figure. What will be potential difference between A and B in the steady state



(a) ε

(b) $\varepsilon / 2$

(c) $\varepsilon / 3$

(d) zero

CHEMISTRY

SECTION – II

This section contains ***30 questions.*** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which ***ONLY ONE*** is correct. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

31. What volume of NO_2 gas at STP is released by the reaction of 10 g Cu with concentrated HNO_3?
(At wt. of $Cu = 63.5g$)
(a) 2 L (b) 4.02 L (c) 6.03 L (d) 7.06 L

32. According to Trouton's rule: $\frac{\Delta H_{vap}}{x} \approx 88\ J\ K^{-1}\ mol^{-1}\ or\ 21\ cal\ K^{-1}\ mol^{-1}$ .What is x, here?
(a) Room temperature (b) Freezing point (c) Boiling point (d) Critical temperature

33. The composition of a sample of wustite is $Fe_{0.95}O$. The percentage of Fe^{3+} in it is:
(a) 5.0 % (b) 10.53 % (c) 10.0 % (d) 5.25 %

34. For C-atom, $1s^2 2s^2 2p_x^1 2p_y^1$, two unpaired electrons are in $l = 1$, the spin angular momentum will be:
(a) $\sqrt{2(2+1)}.\frac{h}{2\pi} + \sqrt{1(1+1)}.\frac{h}{2\pi}$ from $\sqrt{s(s+1)}\frac{h}{2\pi}$ and $\sqrt{1(1+1)}\frac{h}{2\pi}$
(b) $\sqrt{2(2+1)}.\frac{h}{2\pi}$ only
(c) $\sqrt{1(1+1)}.\frac{h}{2\pi}$ only
(d) none of the above

35. The number of spherical nodes, angular nodes and nodal planes for $3p_z$ in proper order are:
(a) 3, 1, 0 (b) 1, 1, 1 (c) 2, 0, 1 (d) 2, 1, 1

36. The stability order of O_2 and its ions is:
(a) $O_2^{2+} > O_2^+ > O_2 > O_2^- > O_2^{2-}$ (b) $O_2^{2+} = O_2^{2-} > O_2^+ = O_2^- > O_2$
(c) $O_2^{2+} = O_2^+ > O_2^{2-} = O_2^- > O_2$ (d) $O_2^{2-} > O_2^- > O_2 > O_2^+ > O_2^{2+}$

37. A gas cylinder contains 14 kg of butane $\left(\Delta H_{combusition} = 2900\,kJ\ mol^{-1}\right)$. If a family needs 2.8×10^4 kJ energy per day and the efficiency of the stove is 80%, how long will the cylinder last?
(a) 56 days (b) 28 days (c) 14 days (d) 20 days

38. For the cell having the reaction,

$Zn(s) + Cu^{2+}(aq) \rightarrow Zn^{2+}(aq) + Cu(s).$

$E^{o}_{Zn^{2+}|Zn} = -0.76\,V$

and $E^{o}_{Cu^{2+}|Cu} = +0.34\,V.$

What is its emf at equilibrium?

(a) 1.10 V (b) – 1.10 V (c) 2.20 V (d) Zero volt

39. One gram of a solute PQ_2 present in 52 g water gives ΔT_b as 0.156 K and one gram of PQ_3 in 52 g water gives ΔT_f as 0.125. At wts. of P and Q are

(a) 12, 15 (b) 15, 30 (c) 30, 15 (d) 32, 16

40. At the equilibrium,

$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$, If pressure is increased:

(a) Yield of SO_3 and the value of equilibrium constant both increase

(b) Yield of SO_3 increases but the value of equilibrium constant does not change

(c) yield of SO_3 increases but the value of equilibrium constant decreases

(d) Yield of SO_3 decreases but the value of equilibrium constant increase

41. For a weak monobasic acid as aqueous solution of concentration C mol L^{-1}, acid dissociation constant K_a and degree of dissociation α, which of the following is correct?

(a) $\left[H^+\right] = Ca$ (b) $\left[H^+\right] = (K_\alpha . C)^{1/2}$

(c) $pH = \frac{1}{2}[pK_a - \log C]$ (d) All of the above

42. $Li^+/Li = -3.05\,V;\ Ba^{2+}/Ba = -2.73V;\ Mg^{2+}/Mg = 2.37V$

The correct order as per reducing power is :

(a) $Li > Ba > Mg$ (b) $Li^+ > Ba^{2+} > Mg^{2+}$ (c) $Mg > Ba > Li$ (d) $Mg^{2+} > Ba^{2+} > Li^+$

43. After a 4 half lives, the amount of a substance left over was 5 g. What was the amount at the start of the experiment?

(a) 20 g (b) 40 g (c) 80 g (d) 100 g

44. For the reaction: $H_2 + I_2 \rightleftharpoons 2HI, +\frac{1}{2}\frac{d[HI]}{dt} = (1.6\times10^{-3})[H_2][I_2] - (2.1\times10^{-2})[HI]^2$

What is the rate constant for the forward reaction?

(a) 1.6×10^{-3} (b) 2.1×10^{-2} (c) $1.6\times10^{-3}\times2.1\times10^{-2}$ (d) $\frac{1.6\times10^{-3}}{2.1\times10^{-2}}$

45. Coagulation of 100 mL. of a negative sol requires 10 mL of 0.5 M NaCl. The coagulation value of NaCl is:

(a) 25 (b) 50 (c) 75 (d) 100

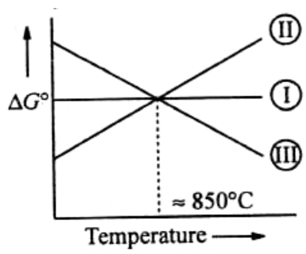
46. First three ionization energies of an element 'A' are 520 kJ mol^{-1}, 7298 kJ mol^{-1} and 11815 kJ mol^{-1} respectively. What is the expected formula of its chloride?

(a) ACl (b) ACl_2 (c) ACl_3 (d) Data is insufficient

47. From the adjacent graphs of oxide formation:

I. $4Fe + 3O_2 \rightarrow 2Fe_2O_3$ II. $2CO + O_2 \rightarrow 2CO_2$ III. $2C + O_2 \rightarrow 2CO$

Which statement is correct?



(a) Fe_2O_3 can be reduced to Fe by CO below 850° C

(b) Fe_2O_3 can be reduced to Fe by C above 850° C

(c) Both (a) and (b) are correct

(d) Both (a) and (b) are wrong

48. Which of the following statements is/are correct?

(a) Li is the weakest reducing alkali metal (b) $LiHCO_3$ is white crystalline solid

(c) It does not react with acetylene (d) Li is the most electropositive alkali metal.

49. In the structure of dichromate ion, the number of Cr–O bonds equal in length are:

(a) 2 and 6 (b) 3 and 5 (c) 2 and 4 (d) 3 and 3.

50. If in the complex $[ML_6]^{3+}$, the metal has oxidation number +3 with $(n-1)d^6$ configuration and L is a strong ligand, the complex is likely to be:

(a) paramagnetic due to 1 unpaired electron (b) paramagnetic due to 4 unpaired electrons

(c) paramagnetic due to 6 unpaired electrons (d) diamagnetic because of no unpaired electron.

51. Which of the following is incorrect for B_2H_6?

(a) sp^2 hybridisation

(b) 3c – 2e bond

(c) extremely high combustibility

(d) formation of an inorganic benzene.

52. Select the incorrect statement(s) :

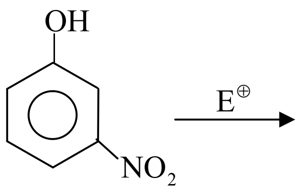
(a) aqua regia has 1 part conc. HNO_3 and 3 parts conc. HCl

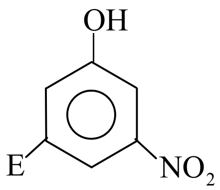
(b) a good sample of bleaching powder has 48% of available chlorine

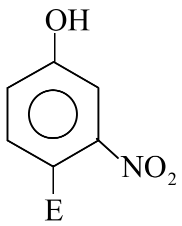
(c) in all oxoacids of halogens, the halogen atom is sp^3 –hybridised

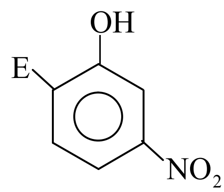
(d) interhalogen compounds are more reactive than halogens.

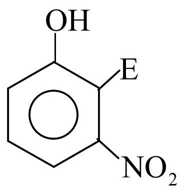
53. Which will not be formed in the following reaction:



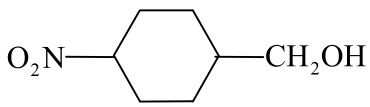
(a) 

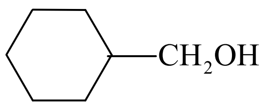
(b) 

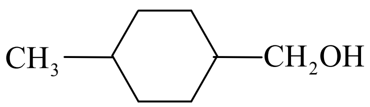
(c) 

(d) 

54. Which of the following will react the most easily with HBr?

(a) 

(b) 

(c) 

(d) all equally.

55. Nucleophilic substitution S_{N^1} is not related with:

(a) 50% retention

(b) 50% inversion

(c) rate $\propto$ [substrate]^2

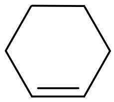
(d) rate $\propto$ [substrate]

56. Which of the following not gives ethane?

(a) $CH_3—CH_2—COONa$ +NaOH/CaO

(b) $CH_3—CH_2—NH_2$ + HI/Red P

(c) $CH_3—CH_2—Mg—Br + CH_3—NH_2$

(d) $CH_2=CH_2$ +  /Pd-373K

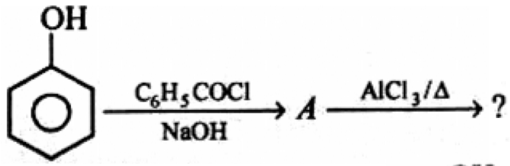
57. Which of the following will give chloroform on heating with a paste of bleaching powder?

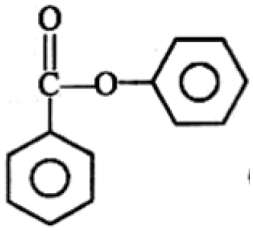
(a) $CH_3—\overset{\overset{\displaystyle O}{||}}{C}—CH_3$

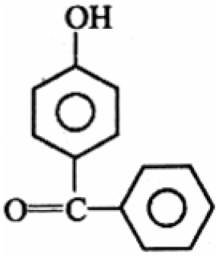
(b) $CH_2Cl—\overset{\overset{\displaystyle O}{||}}{C}—CH_3$

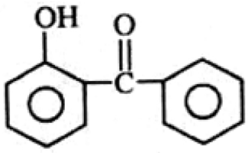
(c) $CHCl_2—\overset{\overset{\displaystyle O}{||}}{C}—CH_3$

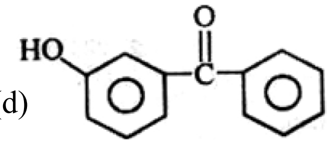
(d) All of the above.

58.  The major product of the reaction finally, is:

(a) 

(b) 

(c) 

(d) 

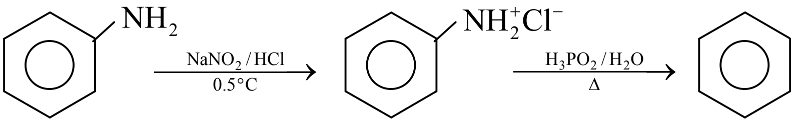
59. $CH_3—CH_2—C\equiv C—CH_3 \xrightarrow{H_2O/H^+-Hg^{2+}}$ .... The major product of this reaction is:

(a) $CH_3—CH_2—CHO$

(b) $CH_3CH_2—COOH$

(c) $CH_3—CH_2—\underset{\underset{\displaystyle O}{||}}{C}—CH_2—CH_3$

(d) $CH_3—CH_2—CH_2—\underset{\underset{\displaystyle O}{||}}{C}—CH_3$

60.  The reaction is known as :

(a) deamination of aniline

(b) Gomberg Bechman reaction

(c) Balz-Schieman reaction

(d) none of these.

MATHS

SECTION – III

This section contains ***30 questions.*** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which ***ONLY ONE*** is correct. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

61. Sum of all the roots of the equation $x^2 - 2x + |x - 1| - 5 = 0$ is

(a) 0 (b) 2 (c) 1 (d) 5

62. If z_1, z_2, z_3, z_4 are points on the circle $|z| = 1$ such that $z_1 + z_2 + z_3 + z_4 = 0$ and the value of the expression $|z_1 - z_2|^2 + |z_2 - z_3|^2 + |z_3 - z_4|^2 + |z_4 - z_1|^2$ is least then,

(a) $z_1 + z_3 = 0$ and $z_2 + z_4 = 0$ (b) $z_1 + z_2 = z_3 + z_4$

(c) z_1, z_2, z_3, z_4 must be real (d) $\dfrac{z_1}{z_3} = \dfrac{z_2}{z_4}$

63. If $T_r = \dfrac{r}{r^4 + 4}$ and $S_n = \sum_{r=1}^{n} t_r$, then the value of $37S_5 - \dfrac{7}{26}$ is equal to

(a) 13 (b) 25 (c) $\dfrac{1}{2}$ (d) $\dfrac{7}{26}$

64. Number of distinct terms in the expansion of $(x + y + z + w)^{50}$ is equal to

(a) ${}^{53}C_3$ (b) 51 (c) 51^2 (d) ${}^{51}C_3$

65. If $y = f(x)$ is symmetric about the lines $3x + 4y + 1 = 0$ and $4x - 3y - 7 = 0$, then it must be symmetric about

(a) (1, 1) (b) (1, –1) (c) (0, 0) (d) (1, 0)

66. If $\sqrt{1-x^6} + \sqrt{1-y^6} = a\left(x^3 - y^3\right)$ and $\dfrac{dy}{dx} = f\left(x, y\right)\sqrt{\dfrac{1-y^6}{1-x^6}}$ then,

(a) $f\left(x, y\right) = \dfrac{y}{x}$ (b) $f\left(x, y\right) = \dfrac{x^2}{y^2}$ (c) $f\left(x, y\right) = \dfrac{2y^2}{x^2}$ (d) $f\left(x, y\right) = \dfrac{y^2}{x^2}$

67. If with standard notations t_1, t_2, t_3, t_4 are four co–normal points on the hyperbola $xy = c^2$ then the orthocentre of the triangle formed by joining the points t_1, t_2, t_3 is given by

(a) (0, 0)
(b) $\left(ct_4, \frac{c}{t_4}\right)$
(c) $\left(c(t_1+t_2+t_3), \frac{c}{t_1+t_2+t_3}\right)$
(d) $\left(\frac{c}{t_4}, ct_4\right)$

68. If S be the focus of a parabola and PQ be the focal chord, such that $SP = 3$ and $SQ = 6$, then the length of latus rectum of the parabola is

(a) 4
(b) 2
(c) 8
(d) 16

69. If a circle having centre at (α, β) cut the circles $x^2 + y^2 - 2x - 2y - 7 = 0$ and $x^2 + y^2 + 4x - 6y - 3 = 0$ orthogonally, then $\left|\frac{3}{4}\alpha - \frac{\beta}{2}\right|$ is equal to

(a) 1
(b) $\frac{1}{2}$
(c) $\frac{1}{4}$
(d) 0

70. Two circles with radii 'r_1' and 'r_2', $r_1 > r_2 \geq 2$, touch each other externally. If 'θ' be the angle between the direct common tangents, then

(a) $\theta = \sin^{-1}\left(\frac{r_1+r_2}{r_1-r_2}\right)$
(b) $\theta = 2\sin^{-1}\left(\frac{r_1-r_2}{r_1+r_2}\right)$
(c) $\theta = \sin^{-1}\left(\frac{r_1-r_2}{r_1+r_2}\right)$
(d) none of these

71. A unit vector in xy – plane which makes an angle of 45° with the vector $\vec{i} + \vec{j}$ and an angle of 60° with the vector $\overrightarrow{3i} - \overrightarrow{4j}$ is

(a) $\vec{i}$
(b) $\frac{\vec{i}+\vec{j}}{\sqrt{2}}$
(c) $\frac{\vec{i}-\vec{j}}{\sqrt{2}}$
(d) none of these

72. Let $(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2} + \ldots x$, $n \in R$. Then the sum of the series $3 + \frac{8}{3} + \frac{80}{3^3} + \frac{240}{3^4} + \ldots$ is

(a) 9
(b) 27
(c) 12
(d) 101

73. There are three points (a, x), (b, y) and (c, z) such that the straight lines joining any two of them are not equally inclined to the coordinate axes where $a, b, c, x, y, z \in R$.

If $\begin{vmatrix} x+a & y+b & z+c \\ y+b & z+c & x+a \\ z+c & x+a & y+b \end{vmatrix} = 0$ and $a + c = -b$, then $x, -\frac{y}{2}, z$ are in

(a) A.P. (b) G.P. (c) H.P. (d) none of these

74. If at $x = 1$, $y = 2x$ is tangent to the parabola $y = ax^2 + bx + c$, then respective values of a, b, c are

(a) $\frac{1}{2}, 1, \frac{1}{2}$ (b) $1, \frac{1}{2}, \frac{1}{2}$ (c) $\frac{1}{2}, \frac{1}{2}, 1$ (d) none of these

75. If $f'\left(x^2 - 4x + 3\right) > 0, \forall x \in (2, 3)$; then $f(\sin x)$ is increasing on

(a) $\bigcup_{n \in I}\left(2n\pi, (4n+1)\frac{\pi}{2}\right)$ (b) $\bigcup_{n \in I}\left((4n-1)\frac{\pi}{2}, 2n\pi\right)$

(c) R (d) none of these

76. Coordinates of the point on the straight line $x + y = 4$, which is nearest to the parabola $y^2 = 4(x - 10)$ is

(a) $\left(\frac{17}{2}, -\frac{9}{2}\right)$ (b) (2, 2) (c) $\left(\frac{3}{2}, \frac{5}{2}\right)$ (d) none of these

77. If $\bar{a}, \bar{b}, \bar{c}$ be three vectors of magnitude $\sqrt{3}$, 1, 2 such that $\bar{a} \times (\bar{a} \times \bar{c}) + 3\bar{b} = 0$, if θ is the angle between $\bar{a}$ and $\bar{c}$, then $\cos^2 \theta$ is equal to

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) none of these

78. If $\Delta(x) = \begin{vmatrix} e^x & \sin 2x & \tan x^2 \\ \ln(1+x) & \cos x & \sin x \\ \cos x^2 & e^x - 1 & \sin x^2 \end{vmatrix} = A + Bx + Cx^2 + \ldots..,$ then B is equal to

(a) 0 (b) 1 (c) 2 (d) none of these

79. If the function $f : [2, \infty) \to [1, \infty)$ is defined by $f(x) = 3^{x(x-2)}$, then $f^{-1}(x)$ is

(a) $1 + \sqrt{1 + \log_3 x}$ (b) $1 - \sqrt{1 + \log_3 x}$ (c) $1 + \sqrt{1 - \log_3 x}$ (d) does not exist

80. The number of real solutions of the equations $\tan^{-1}\sqrt{x^2-3x+2}+\cos^{-1}\sqrt{4x-x^2-3}=\pi$

(a) one (b) two (c) zero (d) infinite

81. Let $x=\left(5\sqrt{2}+7\right)^{19}$, then $x\{x\}$ ($\{.\}$ denotes the fractional part of x) is equal to

(a) 2^{19} (b) 3^{19} (c) 0 (d) 1

82. A flagstaff stands vertically on a pillar, the height of the flagstaff being double the height of the pillar. A man on the ground at a distance finds that both the pillar and the flagstaff subtend equal angles at his eyes. The ratio of the height of the pillar and the distance of the man from the pillar, is

(a) $\sqrt{3}:1$ (b) $1:3$ (c) $1:\sqrt{3}$ (d) $\sqrt{3}:2$

83. $\frac{1}{x}=\frac{2e}{3!}+\frac{4e}{5!}+\frac{6e}{7!}+...\infty$, then find $\int_0^x f(y)\log_y x\,dy,\ y>1$

(a) $\frac{[f(e)]^2}{2}$ (b) $\frac{\left[f\left(\frac{1}{e}\right)\right]^2}{2}$ (c) $\frac{[f(e^2)]^2}{2}$ (d) None of these

84. If lines $x=y=z$, $x=\frac{y}{2}=\frac{z}{3}$ and third line passing through (1, 1, 1) form a triangle of area $\sqrt{6}$ units then point of intersection of third line with second line will be

(a) (1, 2, 3) (b) (2, 4, 6) (c) $\left(\frac{4}{3},\frac{8}{3},\frac{12}{3}\right)$ (d) none of these

85. If $A(\bar{a})$, $B(\bar{b})$ and $C(\bar{c})$ be vertices of a triangle whose circumcentre is the origin, then orthocentre is given by

(a) $\frac{\bar{a}+\bar{b}+\bar{c}}{3}$ (b) $\frac{\bar{a}+\bar{b}+\bar{c}}{2}$ (c) $\bar{a}+\bar{b}+\bar{c}$ (d) none of these

86. Let $f(x)=\max\{\tan x, \cot x\}$. Then number of roots of the equation $f(x)=\frac{1}{\sqrt{3}}$ in $(0, 2\pi)$ is

(a) 2 (b) 4 (c) 0 (d) infinite

87. If S denote the sum to infinity and S_n the sum of n terms of the series $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+...$ such that $S-S_n<\frac{1}{300}$, then the least value of n is

(a) 4 (b) 5 (c) 6 (d) 7

88. If $f(x)=\int_0^4 e^{|t-x|}dt\,(0\le x\le 4)$, the maximum value of $f(x)$ is

(a) e^4-1 (b) $2(e^2-1)$ (c) e^2-1 (d) none of these

89. Let x, y $\in$ [0, 10], then number of solutions (x, y) of the inequation $3^{\sec^2 x-1}\sqrt{9y^2-6y+2}\le 1$ is

(a) 4 (b) 2 (c) 1 (d) infinite

90. A triangle is inscribed in a circle. The vertices of the triangle divide the circle in to three arcs of length 3, 4 and 5 units, then area of the triangle is equal to,

(a) $\frac{9\sqrt{3}\left(1+\sqrt{3}\right)}{\pi^2}$ (b) $\frac{9\sqrt{3}\left(\sqrt{3}-1\right)}{\pi^2}$ (c) $\frac{9\sqrt{3}\left(1+\sqrt{3}\right)}{2\pi^2}$ (d) $\frac{9\sqrt{3}\left(\sqrt{3}-1\right)}{2\pi^2}$

SOLUTION OF AITS JEE (MAIN) FULL TEST – 7

PHYSICS

1. **(a)**

Avg. speed $3 = \dfrac{\sqrt{(v_r t)^2 + (v_{mr} t)^2}}{t}$

$\Rightarrow v_r^2 + 5 = 9 \;\; \Rightarrow v_r = 2\,\text{m/s}$

2. **(b)**

$$v_{\text{avg}} = \frac{\frac{1}{2} \times \frac{1}{2} \times v + \frac{1}{2} \times v}{t} = \frac{3v}{4}$$

3. **(c)**

$$\frac{m}{2} g - T = \frac{m}{2} a \qquad \ldots\text{(i)}$$

$$T \cos 60° = \frac{ma}{\cos 60°} \qquad \ldots\text{(ii)}$$

Solving (i) and (ii) acceleration of ring = $\dfrac{2g}{9}$

4. **(b)**

Work done by all the forces on the block equal to change in kinetic energy.

5. **(d)**

6. **(c)**

No effect of 'a' and 'g' on time period of spring pendulum.

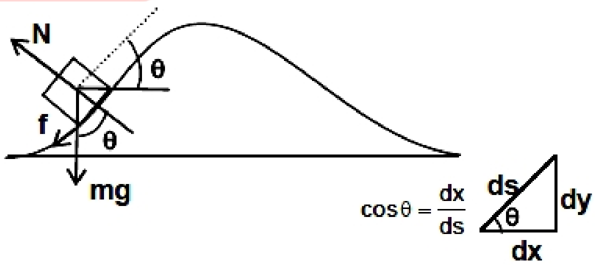
7. **(a)**

$$\frac{1}{\lambda_{3-1}} = R\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = \frac{8R}{9}$$

$$\frac{1}{\lambda_{2-1}} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3R}{4}$$

$$\Rightarrow \frac{\lambda_{3-1}}{\lambda_{2-1}} = \frac{27}{32}$$

8. **(c)**



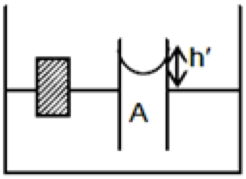
Work done by friction $= \int \vec{F} \cdot \vec{ds} = \int_0^x \mu mg \cos\theta \dfrac{dx}{\cos\theta}$

$= \mu mg\, x = 20\,\text{J}$

9. **(a)**

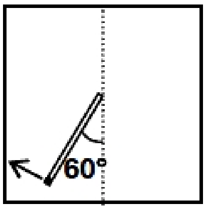
There will be no change.

$\therefore \; h' = h$



10. **(c)**

Conservation of energy $\frac{1}{2}mv^2 - mg\frac{\ell}{2} = mg\frac{\ell}{2} + m\sqrt{3}\left(2\ell\frac{\sqrt{3}}{2}\right)g$



$v = \sqrt{8g\ell}$

11. **(d)**

Potential across capacitor is zero, hence energy stored is zero.

12. **(c)**

$v_A - v_B = \int E.dx = 10(1) = 10 \text{ volt}$

13. **(c)**

$Z = \sqrt{R^2 + X^2} = \sqrt{9 + X^2}$

but $\cos\phi = \frac{R}{Z} = \frac{3}{5}$

$X = 4\Omega.$

14. **(a)**

$\varepsilon = \frac{\sigma}{\varepsilon_0}$

$v_A - v_B = \frac{\sigma}{\varepsilon_0}(b) - \frac{\sigma}{\varepsilon_0}(a)$

$\therefore W = Q(v_A - v_B) = \frac{Q\sigma}{\varepsilon_0}(b - a)$

15. **(b)**

$\omega = 0 + 1 \times 10 = 10 \text{ rad/sec}^2$

$\therefore \; v = r\omega = 1 \times 10 = 10 \text{ m/s}$

$\bar{B} = \frac{\mu_0}{4\pi}\frac{q(\bar{v} \times \bar{r})}{r^2} \Rightarrow |\bar{B}| = \frac{\mu_0 qv}{4\pi r^2}$

$B = \frac{10^{-7} \times 0.1 \times 10}{(1)^2} = 10^{-7} T$

16. **(d)**

$P = \sigma A T^4$

$\Rightarrow \frac{P_A}{P_B} = \frac{A_B}{A_A}\left(\frac{T_B}{T_A}\right)^4 \Rightarrow 16 = \frac{2}{1}\left(\frac{T_B}{T_A}\right)^4 \Rightarrow T_B = T_A(8)^{1/4}$

Since $\lambda_m T = \text{constant}$, $\frac{\lambda_A}{\lambda_B} = \frac{T_B}{T_A} = (8)^{1/4} \Rightarrow \lambda_B = \frac{\lambda_A}{(8)^{1/4}} = \frac{5000}{(8)^{1/4}}$ Å

17. **(a)**

For adiabatic process, $TV^{\gamma-1} = \text{constant}$

$$T\left(\frac{m}{\rho}\right)^{\gamma-1} = \text{constant}$$

$$\frac{T}{\rho^{\gamma-1}} = \text{constant}$$

$$\rho \propto T^{1/(\gamma-1)} \Rightarrow \frac{1}{\gamma-1} = 3 \Rightarrow \gamma = 4/3$$

$$f = \frac{2}{\gamma-1} = \frac{2}{\left(\frac{4}{3}-1\right)} = 6$$

18. **(c)**

$$P(x, y),\ A(-3a, 0),\ B = (3a, 0)$$

$$V_{PA} = \frac{1}{4\pi\varepsilon_0}\frac{-2Q}{PA},\ V_{PB} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{PB}$$

According to equation

$$\frac{1}{4\pi\varepsilon_0}\left(\frac{-2Q}{PA}\right) + \frac{1}{4\pi\varepsilon_0}\frac{Q}{PB} = 0$$

$$\frac{2}{PA} = \frac{1}{PB} \Rightarrow 4PB^2 = PA^2$$

$$(x-5a)^2 + y^2 = (4a)^2$$

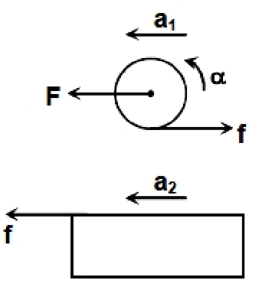
19. **(a)**

$$i = \sqrt{5}A$$

$$\frac{q_m^2}{2C} = \frac{q^2}{2C} + \frac{1}{2}Li^2 \Rightarrow q_{\max} = 6C$$

20. **(c)**

21. **(a)**



$$f = ma_2$$

$$\alpha = \frac{\tau}{I} = \frac{2f}{mR}$$

$$\Rightarrow a_2 = F/4\text{m},\ f = F/4$$

22. **(d)**

The process is equivalent to $TV^{1/2} = C$

Compare with $TV^{x-1} = C \Rightarrow x = 3/2$

$$\Rightarrow C = \frac{R}{\gamma - 1} + \frac{R}{1 - X} = \frac{R}{2/3} + \frac{R}{1 - (3/2)} = \frac{3}{2}R - 2R = -\frac{1}{2}R$$

23. **(a)**

Intensity will be highest at the nearest point.

24. **(c)**

25. **(d)**

26. **(b)**

27. **(b)**

28. **(d)**

$$\tan\theta = \frac{u^2}{Rg} \Rightarrow R = \frac{u^2}{g\tan\theta} = \frac{100}{10 \times \sqrt{3}} = \frac{10}{\sqrt{3}}$$

29. **(c)**

30. **(d)**

CHEMISTRY

31. **(d)**

$$\underset{63.5\text{g}}{\text{Cu}} + 4\text{HNO}_3 \rightarrow \text{Cu(NO}_3) + \underset{2\times 22.4\,\text{L at STP}}{2\text{NO}_2} + 2\text{H}_2\text{O}$$

$$10\text{g Cu} \equiv \frac{2 \times 22.4 \times 10}{63.5\text{g}}\text{L NO}_2 \text{ at STP}$$

$= 7.06$ L

32. **(c)**

33. **(b)**

Let the number of Fe^{3+} ions be x. So, the number of Fe^{2+} ions is $95 - x$, for $Fe_{95}O_{100}$

Balancing the charge,

$$3x + 2(95 - x) = 200$$

$$\Rightarrow \quad x = 200 - 190 = 10$$

Percentage of $Fe^{3+} = \frac{10}{95} \times 100 = 10.53\%$

34. **(c)**

In $\sqrt{s(s+1)} \times \frac{h}{2\pi}$ replace s by $\frac{2}{2}$, *i.e.*, 1, for two unpaired electrons.

35. **(b)**

Spherical nodes for $3p_z = n-l-1 = 3-1-1 = 1$

Angular nodes for $3p_z = l = 1$

Nodal planes for $3p_z = l = 1$

36. **(a)**

Bond order are: $O_2^{2+} = 3.0$, $O_2^{+} = 2.5, O_2 = 2.0, O_2^{-} = 1.5$ and $O_2^{2-} = 1.0$

37. **(d)**

For 80% efficiency the gas used = $14 \times \frac{80}{100} = 11.2\,kg$

58 g fuel = 2900 kJ

11.2 kg fuel = $\frac{2900 \times 11.2 \times 1000}{58}$ kJ = 560000 kJ

2.8×10^4 kJ energy = 1 day

560000 kJ energy = $\frac{560000}{2.8 \times 10000}$ = 20 days.

38. **(d)**

emf of a cell at equilibrium is zero but emf is never zero except for concentration cell.

39. **(d)**

$$M_{(PQ_2)} = \frac{1000 k_b w_B}{\Delta T_b w_A} = \frac{1000 \times 0.52 \times 1}{0.156 \times 52} = 64.1$$

$$M_{(PQ_3)} = \frac{1000 \times 0.52 \times 1}{0.125 \times 52} = 80$$

If at. wts. of P and Q are x and y, x + 2y = 64.1 and x + 3y = 80

$\Rightarrow$ x = 32 and y = 16

40. **(b)**

Since, the number of moles of moles of gaseous substances on product side is less, increase in pressure will increase the yield. Equilibrium constant will not change because it depends only on temperature (for a specific reaction).

41. **(d)**

42. **(a)**

In this question, reduction potentials are in the order Li < Ba < Mg. Hence, the order of reducing power is Li > Ba > Mg.

43. **(c)**

Amount left $[A] = \frac{[A]_0}{2^n}$

$$[A]_0 = 5 \times 2^4 = 80\text{ g}$$

44. **(a)**

Compare with

$$+\frac{1}{2}\frac{d[HI]}{dt}=k_1[H_2][I_2]-k_2[HI]^2, \qquad k_1 = 1.6 \times 10^{-3}.$$

45. **(b)**

10 mL of 0.5 M NaCl

= 10 × 0.5 millimoles of NaCl

= 5 millimoles of NaCl

100 mL of negative sol requires NaCl

= 5 millimols

1000 mL (*i.e.*, 1 L) sol requies NaCl

$$=\frac{5\times1000}{100}=50 \text{ millimoles.}$$

46. **(a)**

A sudden jump from first to second IE shows the valency of the atom 'A' to be 1. So, the formula of chloride is ACl

47. **(c)**

Graphs II is below graph I of formation of Fe_2O_3 upto 850°C and graph III is below the graph I of formation of Fe_2O_3 after 850°C.

48. **(c)**

49. **(a)**

50. **(d)**

C.N 6 shows $d^2\ sp^3$ – hybridisation. Strong ligand will pair up (n–1) d^6 electrons to vacate two d-orbitals.

51. **(a)**

52. **(b)**

53. **(a)**

The *o*/ *p*-directing group 'OH' dominates over the *m*-directing group 'NO_2'

54. **(c)**

+*I* effect of CH_3 — increases the basic character of — OH group and makes its reaction easier with HBr

55. **(c)**

56. **(b)**

57. **(d)**

58. **(c)**

59. **(d)**

Methyl ketone being thermodynamically more stable, is the major product.

60. **(a)**

NH_2 group of aniline has been removed.

MATHS

61. **(b)**

Given equation can be written as $|x-1|^2+|x-1|-6=0$

$t^2+t-6=0 \Rightarrow t=2$ or $-3 \Rightarrow |x-1|=2 \Rightarrow x-1=\pm 2$ or $x=3,\,-1$

62. **(a)**

$|z_1-z_2|^2+|z_2-z_3|^2+|z_3-z_4|^2+|z_4-z_1|^2$

$=\left(2|z_1|^2+|z_2|^2+|z_3|^2+|z_4|^2\right)+|z_1+z_3|^2+|z_2+z_4|^2 \geq 8+|z_1+z_3|^2+|z_2+z_4|^2$

63. **(a)**

$$T_r=\frac{r}{r^4+4}=\frac{r}{\left(r^2+2\right)^2-4r^2}=\frac{r}{\left(r^2+2r+2\right)\left(r^2-2r+2\right)}$$

$$=\frac{1}{4}\left[\frac{1}{r^2-2r+2}-\frac{1}{r^2+2r+2}\right]=\frac{1}{4}\left[\frac{1}{(r-1)^2+1}-\frac{1}{(r+1)^2+1}\right]$$

$$\Rightarrow S_n=\frac{1}{4}\left[1+\frac{1}{2}-\frac{1}{n^2+1}-\frac{1}{(n+1)^2+1}\right]$$

$$\Rightarrow S_5=\frac{1}{4}\left[1+\frac{1}{2}-\frac{1}{26}-\frac{1}{37}\right]=\frac{1}{4}\left[\frac{38\times 37-26\times 1}{26\times 37}\right]=\frac{1}{4}\times\frac{1380}{26\times 37}=\frac{345}{26\times 37}$$

$$\Rightarrow 37S_5-\frac{7}{26}=\frac{338}{26}=13$$

64. **(a)**

Number of terms = Number of non–negative integral solution of the equation $p+q+r+s=50$

$={}^{50+4-1}C_{50}={}^{53}C_{50}={}^{53}C_3$

65. **(b)**

If a function is symmetric about two mutually perpendicular lines, it must be symmetric about their point of intersection.

66. **(b)**

$$-\frac{x^5}{\sqrt{1-x^6}}-\frac{y^5}{\sqrt{1-y^6}}\frac{dy}{dx}=a\left(x^2-y^2\frac{dy}{dx}\right)$$

$$\Rightarrow\left(ay^2-\frac{y^5}{\sqrt{1-y^6}}\right)\frac{dy}{dx}=ax^2+\frac{x^5}{\sqrt{1-x^6}}$$

$$\Rightarrow\frac{dy}{dx}=\frac{\sqrt{1-y^6}}{\sqrt{1-x^6}}\times\frac{x^2\left(a\sqrt{1+x^6}+x^3\right)}{y^2\left(a\sqrt{1-y^6}-y^3\right)} \qquad \ldots(1)$$

Also, $\frac{(1-x^6)-(1-y^6)}{\sqrt{1-x^6}-\sqrt{1-y^6}}=a(x^3-y^3)$ or $x^3+y^3=a\left(\sqrt{1-y^6}-\sqrt{1-x^6}\right)$

$\Rightarrow a\sqrt{1-y^6}-y^3=a\sqrt{1-x^6}+x^3$

Hence, from equation (1), $\frac{dy}{dx}=\frac{\sqrt{1-y^6}}{\sqrt{1-x^6}}\times\frac{x^2}{y^2}$

67. **(b)**

For a triangle formed by joining any three co–normal points, orthocentre consides with the fourth point.

68. **(c)**

HM of SP and SQ = $\frac{2(3)(6)}{3+6}=4$ semi latus rectum.

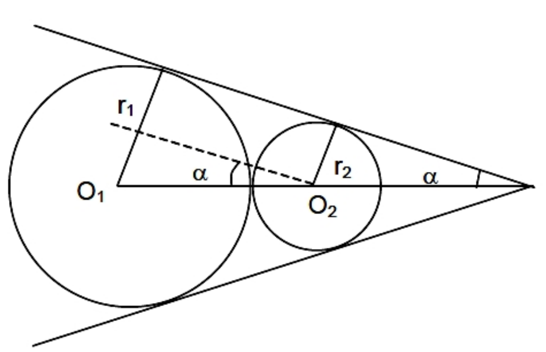
69. **(b)**

If a circle cuts two circles orthogonally, radical axis of the two circles pass through the centre of the first circle

Here radical axis of given circles is $6x-4y+4=0$ or $3x-2y+2=0$

Hence $3\alpha-2\beta+2=0 \Rightarrow \frac{3}{4}\alpha-\frac{\beta}{2}=-\frac{1}{2}$

70. **(b)**



$\sin\alpha=\frac{r_1-r_2}{r_1+r_2}$

$\Rightarrow \theta=2\sin^{-1}\left(\frac{r_1-r_2}{r_1+r_2}\right)$

71. **(d)**

Let $\vec{r}=a\vec{i}+b\vec{j} \Rightarrow a^2+b^2=1$ ...(1)

Also,

$\cos 45^\circ=\frac{\vec{r}\cdot(\vec{i}+\vec{j})}{|\vec{r}||\vec{i}+\vec{j}|} \Rightarrow a+b=1$ ...(2)

$\cos 60^\circ=\frac{\vec{r}\cdot(3\vec{i}-4\vec{j})}{|\vec{r}||3\vec{i}-4\vec{j}|} \Rightarrow 3a-4b=\frac{5}{2}$ ...(3)

There exists no real values of a and b satisfying (1), (2) and (3)

Hence no such unit vector exists

72. **(b)**

$$(1+x)^n = 3+\frac{8}{3}+\frac{80}{3^3}+\frac{240}{3^4}+\ldots = 1+nx+\frac{n(n-1)x^2}{2}+\ldots$$

On comparison, $n = -3$ and $x = -\frac{2}{3}$

73. **(a)**

From the given conditions, $\frac{y-x}{b-a} \neq \pm 1,\ \frac{z-y}{c-b} \neq \pm 1,\ \frac{z-x}{c-a} \neq \pm 1$

$\Rightarrow x+a \neq y+b \neq z+c$ The determinant is a symmetric one. The determinant will be equal to zero if

$x+a+y+b+z+c=0$

but $a + b + c = 0$ (given)

$\Rightarrow x+y+z=0 \Rightarrow x+z=2\left(-\frac{y}{2}\right) \Rightarrow x, -\frac{y}{2}, z$ are in A.P.

74. **(a)**

For $x = 1$, $y = a + b + c$

Tangent at $(1, a + b + c)$ is $\frac{1}{2}(y+a+b+c) = ax+\frac{b}{2}(x+1)+c$

Comparing with $y = 2x$, $c = a$, $b = 2(1 - a)$

75. **(b)**

$x \in (2, 2) \Rightarrow -1 < x^2 - 4x + 3 < 0$, so $f(x)$ is increasing in $(-1, 0)$

$\Rightarrow f(\sin x)$ is increasing on $\bigcup_{n\in I}\left((4n-1)\frac{\pi}{2}, 2n\pi\right)$

76. **(a)**

Let point P on the straight line $x + y = 4$ be $(m, 4 - m)$, this will be nearest to the parabola if $\perp$ at this point to the straight line becomes normal to the parabola.

Let it is normal at $x - 10 = t^2$, $y = 2t$

Perpendicular to $x + y = 4$ at $(m, 4 - m)$ is $y - (4 - m) = (x - m)$ …(1)

Normal at parabola at $(t^2 + 10, 2t)$ is $y + t(x - 10) = 12t + t^3$ …(2)

(1) and (2) are same $\Rightarrow t = -1,\ m = \frac{17}{2}$ so required point is $\left(\frac{17}{2}, -\frac{9}{2}\right)$

77. **(a)**

$|\bar{a}\times(\bar{a}\times\bar{c})| = |3\bar{b}| = 3|\bar{b}|$

$|\bar{a}|\cdot|(\bar{a}\times\bar{c})|\sin\frac{\pi}{2} = 3.1 \Rightarrow 3 = |\bar{a}|.(\|\bar{a}|\ |\bar{c}\|\sin\theta) \Rightarrow 3 = 3.2\sin\theta \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \cos^2\theta = \frac{3}{4}$

78. **(a)**

$$\Delta'(x)=\begin{vmatrix} e^x & 2\cos 2x & 2x\sec^2 x^2 \\ \ln(1+x) & \cos x & \sin x \\ \cos x^2 & e^x-1 & \sin x^2 \end{vmatrix}+\begin{vmatrix} e^x & \sin x & \tan x^2 \\ \frac{1}{(1+x)} & -\sin x & \cos x \\ \cos x^2 & e^x-1 & \sin x^2 \end{vmatrix}+\begin{vmatrix} e^x & \sin 2x & \tan x^2 \\ \ln(1+x) & \cos x & \sin x \\ -2x\sin x^2 & e^x & 2x\cos x^2 \end{vmatrix}$$

$= B+2Cx+\ldots$

Put $x=0,\ B=\begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix}+\begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \\ 1 & 0 & 0 \end{vmatrix}+\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{vmatrix}=0$

79. **(a)**

Let $g(x)$ be the inverse of f, then $f(g(x)) = x$

$\Rightarrow 3^{g(x)(g(x)-2)}=x \Rightarrow (g(x))^2-2g(x)-\log_3 x=0$

$\Rightarrow g(x)=\dfrac{2\pm\sqrt{4+4\log_3 x}}{2}=1\pm\sqrt{1+\log_3 x}$

Since $g:[1,\infty]\to[2,\infty]$

So $g(x)=1+\sqrt{1+\log_3 x}$

80. **(c)**

Since $\sqrt{x^2-3x+2}\ge 0 \Rightarrow 0\le \tan^{-1}\sqrt{x^2-3x+2}<\dfrac{\pi}{2}$

Since $\sqrt{4x-x^2-3}\ge 0 \Rightarrow 0<\cos^{-1}\sqrt{4x-x^2-3}\le\dfrac{\pi}{2}$

$\Rightarrow 0<\text{L.H.S}<\pi \Rightarrow$ The given equation has no solution

81. **(d)**

Let $f=\left(5\sqrt{2}-7\right)^{19}$

$x-f=$ an integer $\Rightarrow [x]+\{x\}-f=$ an integer

$\Rightarrow \{x\}-f=$ an integer, but $-1<\{x\}-f<1\Rightarrow\{x\}=f$

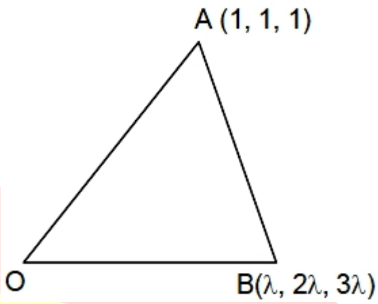
So, $x\{x\}=x.f=1^{19}=1$

82. **(c)**

83. **(d)**

84. **(b)**

Let any point on second line be $(\lambda, 2\lambda, 3\lambda)$



$$\cos\theta = \frac{6}{\sqrt{42}} \sin\theta = \frac{6}{\sqrt{42}}$$

$$\Delta_{OAB} = \frac{1}{2}(OA).OB\sin\theta = \frac{1}{2}\sqrt{3}.\lambda\sqrt{14}\times\frac{6}{\sqrt{42}} = \sqrt{6}$$

$$\Rightarrow \lambda = 2$$

So, B is $(2, 4, 6)$

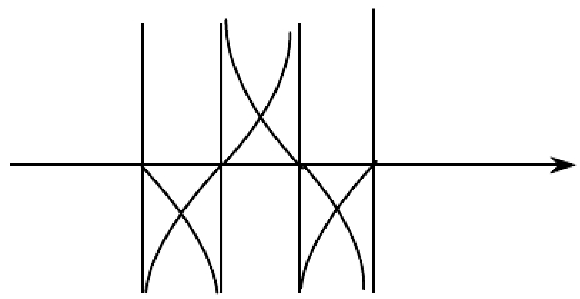
85. **(c)**

Centroid of triangle will be $\dfrac{\bar{a}+\bar{b}+\bar{c}}{3}$

Now line joining the orthocentre and the circumcentre is divided by centroid in 2 : 1 ratio internally, so orthocentre will be $\bar{a}+\bar{b}+\bar{c}$.

86. **(c)**

If we draw the graph of tan x and cot x, we observes that range of f (x) is $[-1, 0) \cup [1, \infty)$



So f (x) = $\dfrac{1}{\sqrt{3}}$ does not have any root

87. **(c)**

88. **(a)**

$$f(x) = \int_0^x e^{|t-x|}dt + \int_x^4 e^{|t-x|}dt = \int_0^x e^{x-t}dt + \int_x^4 e^{t-x}dt = -e^{x-t}\Big|_0^x + e^{t-x}\Big|_x^4 = e^x + e^{4-x} - 2$$

$$f'(x) = e^x - e^{4-x} = 0 \Rightarrow x = 4 - x \Rightarrow x = 2$$

$f(0) = f(4) = e^4 - 1,\ f(2) = 2(e^2 - 1)$, so maximum value of $f(x)$ is $e^4 - 1$.

89. **(a)**

Given inequation can be rewritten as $3^{\sec^2 x}\sqrt{y^2 - \frac{2y}{3} + \frac{2}{9}} \le 1$

$$\Rightarrow 3^{\sec^2 x}\sqrt{\left(y-\frac{1}{3}\right)^2 + \frac{1}{9}} \le 1$$

Now, $3^{\sec^2 x} \ge 3$ and $\sqrt{\left(y-\frac{1}{3}\right)^2 + \frac{1}{9}} \ge \frac{1}{3}$

So, we should have $\sec^2 x = 1,\ y = \frac{1}{3} \Rightarrow x = 0, \pi, 2\pi, 3\pi$

90. **(a)**