NTPL
ALL INDIA TEST SERIES

CODE - A

TEST ID 001921

JEE (Main) - 2019

FULL TEST - 6

Time : 3 Hours | | *Maximum Marks : 360*

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

You are not allowed to leave the Examination Hall before the end of the test.

INSTRUCTIONS

A. General :

1. This booklet is your Question Paper containing **90 questions**.
2. The Question Paper **CODE & TEST ID** is printed on the right hand top corner of this booklet. This should be entered on the OMR Sheet.
3. Fill the bubbles completely and properly using a **Blue/Black Ball Point Pen** only.
4. No additional sheets will be provided for rough work.
5. Blank papers, clipboards, log tables, slide rules, calculators, cellular phones, pagers, and electronic gadgets in any form are not allowed to be carried inside the examination hall.
6. The answer sheet, a machine-readable Optical mark recognition sheet (OMR Sheet), is provided separately.
7. DO NOT TAMPER WITH / MUTILATE THE OMR OR THE BOOKLET.
8. Do not break the seals of the question-paper booklet before being instructed to do so by the invigilator.

B. Question paper format & Marking Scheme :

9. The question paper consists of **3 parts** (Physics, Chemistry and Maths).
10. The test is of **3 hours** duration. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct. Each question carries **+4 marks** for correct answer and **-1 mark** for wrong answer.

Name of the Candidate (in Capitals) ____________________

Test Centre ____________________ Centre Code ____________________

Candidate's Signature ____________________ Invigilator's Signature ____________________

PHYSICS

SECTION – I

This section contains ***30 questions.*** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which ***ONLY ONE*** is correct. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

1. A particle starts from rest with an acceleration $2\,\text{ms}^{-1}$. After acquiring maximum velocity it decelerates with $3\,\text{ms}^{-2}$ and covers 1500 m before coming to rest. Find the time for which it has moved.

(a) 50 s (b) 45 s (c) 35 s (d) 42.4 s

2. Two projectiles each of mass m are projected with same velocity v making an angle α and β from the same point in opposite directions. Find the change in their momentum at instant before w.r.t. to the projection.

(a) $2mv\sin(\alpha+\beta)$ (b) $2mv\sin\frac{\alpha+\beta}{2}$ (c) $2mv\cos(\alpha+\beta)$ (d) $2mv\cos\frac{(\alpha+\beta)}{2}$

3. A particle is placed at the top of a sphere of radius r. It is given a little jerk so that it just starts slipping down. Find the depth of point from where it leaves the sphere, from initial level.

(a) $r/2$ (b) $r/3$ (c) $r/4$ (d) r

4. A ball falls on an inclined plane of inclination θ from a height h above the point of impact and makes a perfectly elastic collision. Where will it hit the inclined plane again?

(a) $8h/\sin\theta$ (b) $8h\sin\theta$ (c) $4h\sin\theta$ (d) $4h/\sin\theta$

5. A small solid sphere of radius r rolls down an incline without slipping which ends into a vertical loop of radius R. Find the height above the base so that it just loops the loop

(a) $\frac{5}{2}R$ (b) $\frac{5}{2}(R-r)$ (c) $\frac{25}{10}(R-r)$ (d) $\frac{27}{10}R-\frac{17r}{10}$

6. A Hg drop of radius 1 cm is splitted to 10^6 droplets of equal size. The energy required to split it if the surface tension of Hg is $35\times10^{-3}\,\text{Nm}^{-1}$

(a) $5.35\times10^{-3}\,\text{J}$ (b) $4.35\times10^{-3}\,\text{J}$ (c) $4.35\times10^{-3}\,\text{cal}$ (d) $5.35\times10^{-3}\,\text{cal}$

7. Water from a tap emerges vertically downwards with an initial speed of $1\,\text{ms}^{-1}$. The cross-sectional area of the tap is $10^{-4}\,\text{m}^2$. Assuming pressure to be constant throughout the stream of water and flow to be steady the cross-sectional area 0.15 m below the tap is

(a) $10^{-4}\,\text{m}^2$ (b) $10^{-5}\,\text{m}^2$ (c) $0.5\times10^{-4}\,\text{m}^2$ (d) $0.2\times10^{-4}\,\text{m}^2$

8. A cylinder has an alloy (piston) at a temperature of 20°C. There is all round clearance of 0.05mm between piston and cylinder wall when the internal diameter of the cylinder is exactly 10cm. The temperature at which it will exactly fit into the cylinder is

(a) 220°C (b) 250°C (c) 270°C (d) 290°C

Given expansion coefficient of alloy is $1.6\times10^{-5}/°\text{C}$ and expansion coefficient of cylinder is $1.2\times10^{-5}/°\text{C}$.

9. A bimetallic strip is formed out of two identical strips, one of Cu and the other of brass. The coefficients of linear expansion of the two metals are α_C and α_B. If on heating the temperature of the strip goes up by ΔT and the strip bends to form an arc of radius R, then R is

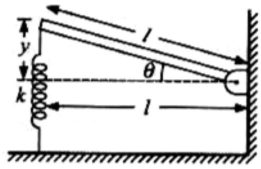
(a) proportional to ΔT (b) inversely proportional to ΔT

(c) proportional to $|\alpha_B - \alpha_C|$ (d) inversely proportional to $|\alpha_B - \alpha_C|$

10. 3 mole of H_2 is mixed with 1 mole of Ne. The specific heat at constant pressure is

(a) $9R/4$ (b) $13R/4$ (c) $9R/2$ (d) $13R/2$

11. In the system shown a long uniform rod is attached at one end to a spring of spring constant k and the other end is hinged. It is displaced slightly and allowed to oscillate. The time period of oscillations is



(a) $2\pi\sqrt{\dfrac{M}{k}}$ (b) $2\pi\sqrt{\dfrac{M}{2k}}$

(c) $2\pi\sqrt{\dfrac{M}{3k}}$ (d) none of these

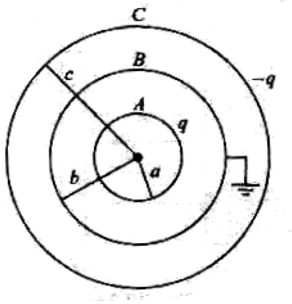
12. Find the speed of sound in a mixture of 1 mole of He and 2 mole of O_2 at 27°C

(a) 480ms^{-1} (b) 621ms^{-1} (c) 401ms^{-1} (d) 601ms^{-1}

13. Charge Q is distributed uniformly on length l of a wire. It is bent in the from of a semicircle. Find the electric field at the centre of the ring.

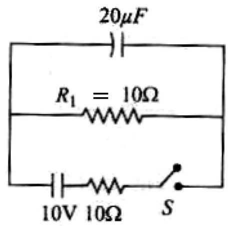
(a) $\dfrac{Q}{\varepsilon_0 l^2}$ (b) $\dfrac{2Q}{\varepsilon_0 l^2}$ (c) $\dfrac{Q}{\varepsilon_0 l}$ (d) $\dfrac{Q}{2\varepsilon_0 l^2}$

14. **A**, **B** and **C** are three concentric spherical shells of radius **a, b** and **c** respectively. The charges on shells **A** and **C** are **q** and **–q** respectively and shell **B** is earthed as shown in figure. The charge on outer surface of **B** is



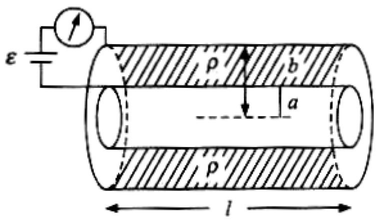
(a) zero
(b) $\mathbf{a/cq}$
(c) $\mathbf{a/bq}$
(d) $\mathbf{\frac{bq}{c}}$

15. The switch S is kept closed for a long time. It is opened at $t=0$. Find the current in R_1 at $t=1\,\text{ms}$.



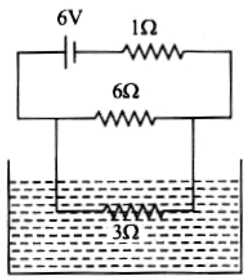
(a) 11.2 mA
(b) 12.4 mA
(c) 13.4 mA
(d) 14.4 mA

16. A hollow cylinder of radius a and b is filled with a material of resistivity ρ. Find the current through ammeter



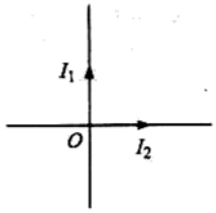
(a) $\frac{\varepsilon\pi\left(b^2-a^2\right)}{\rho l}$
(b) $\frac{\varepsilon\pi l}{\left(\rho \log\frac{b}{a}\right)}$
(c) $\frac{\varepsilon 2\pi l}{\rho \log_e \frac{b}{a}}$
(d) $\frac{\varepsilon 2\pi l}{\rho \log_e \frac{a}{b}}$

17. A 3Ω resistor as shown in figure is dipped into a calorimeter containing H_2O. The thermal capacity of water + calorimeter is $2000\,\text{J/K}$. If the circuit is active for 15 minutes find the rise in temperature of water is



(a) 2.4°C
(b) 2.9°C
(c) 3.4°C
(d) 1.9°C

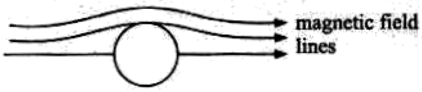
18. Two long wires carrying current are kept crossed (not joined at O). The locus where magnetic field is zero is



(a) $I_1 = \frac{x}{y} I_2$ (b) $I_1 = \frac{y}{x} I_2$

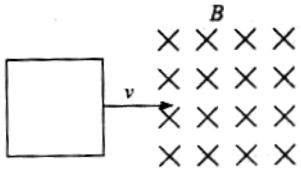
(c) $I_1 = I_2$ (d) $I_1 = -I_2$

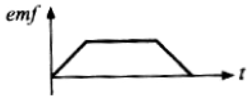
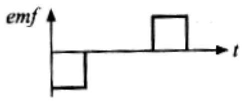
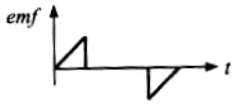
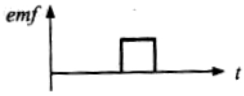
19. The material which shows the effect shown in figure when placed in a uniform magnetic field is called



(a) paramagnetic (b) diamagnetic (c) ferromagnet (d) antiferromagnetic

20. The emf generated in a loop when it enters a magnetic field with a velocity as shown in figure is best depicted in

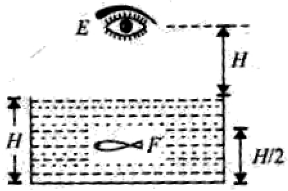


(a)  (b)  (c)  (d) 

21. $I = 6\cos\omega t + 8\sin\omega t$ is applied across a 40Ω resistor. Find the potential difference across the resistor.

(a) 560 V (b) 80 V (c) 320 V (d) 400 V

22. Refractive index of water in the situation shown in figure is μ. Find the distance seen by the fish F of human eye E



(a) $H + \frac{H}{2\mu}$ (b) $\frac{3H}{2\mu}$

(c) $\frac{H}{2} + H\mu$ (d) $\frac{3\mu H}{2}$

23. In a YDSE $\lambda = 500\text{nm}, d = 1\text{mm}, D = 1\text{m}$. The minimum distance from the central maximum for which the intensity is half of maximum intensity is

(a) $2.5\times10^{-4}\,\text{m}$ (b) $5\times10^{-4}\,\text{m}$ (c) $1.25\times10^{-4}\,\text{m}$ (d) $1.67\times10^{-4}\,\text{m}$

24. Anode voltage is at +3V. Incident radiation has frequency $1.4\times10^{15}\,\text{Hz}$ work function of the photocathode is $2.8\,\text{eV}$. The minimum and maximum kE of photoelectrons on anode in eV is

(a) 3, 6 (b) 0, 3 (c) 0, 6 (d) 2.8, 5.8

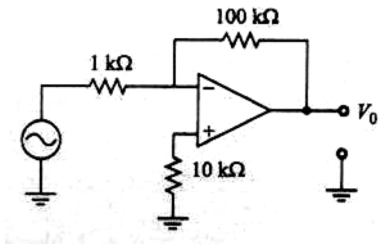
25. An atom initially at an energy level $E=-6.52\text{eV}$, absorbs a photon of wavelength 860nm. What is the internal energy of atom after absorbing photon?

(a) 5.08eV (b) 1.44eV (c) –1.44eV (d) –5.08eV

26. When $_3^7\text{Li}$ $(M_{\text{Li}}=7.016004\,\text{u})$ is bombarded by a proton, 2α-particles result. $(M_{\text{He}}=4.002603\,\text{u})$. Find the reaction energy.

(a) 13.35 MeV (b) 14.85 MeV (c) 16.05 MeV (d) 17.35 MeV

27. The voltage gain of the amplifier shown is



(a) 10

(b) 100

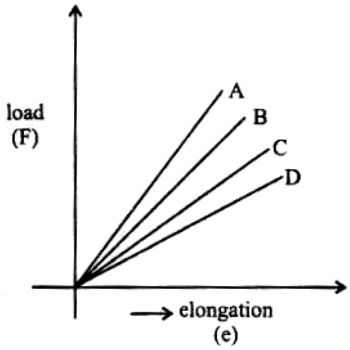
(c) 9.9

(d) 1000

28. When the modulating frequency in an FM system is 500 Hz and the modulating voltage is 2.5V, the modulation index is 50. Find the maximum deviation. Also find the modulation index if modulating signal is $e_m=4.5\sin 400\pi t$.

(a) 25kHz, 225 (b) 25kHz, 50 (c) 25kHz, 45 (d) 25kHz, 125

29. The load verses elongation graph for four wires of the same material is as shown in figure, which of the wire is thickest



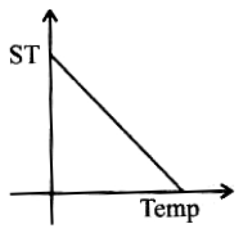
(a) A

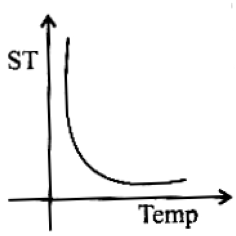
(b) B

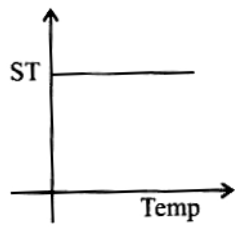
(c) C

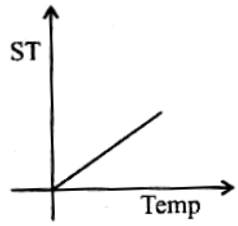
(d) D

30. Which graph represent the variation of surface tension over small temperature ranges for water?

(a) 

(b) 

(c) 

(d) 

CHEMISTRY

SECTION – II

This section contains ***30 questions.*** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which ***ONLY ONE*** is correct. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

31. For the equilibrium, $Ag^+ + 2NH_3 \rightleftharpoons [Ag(NH_3)_2]^+$ $K_1 = 1.8 \times 10^7$ and $Ag^+ + Cl^- \rightleftharpoons AgCl$; $K_2 = 5.6 \times 10^9$ hence, for the equilibrium : $AgCl + 2NH_3 \rightleftharpoons Ag(NH_3)_2{}^+ + Cl^-$ equilibrium constant is :

(a) 0.32×10^{-2} (b) 0.31×10^{-21} (c) 1.01×10^{17} (d) 1.01×10^{-17}

32. 2-Pentyne, on reduction with lithium in liq. NH_3 yields

(a) pentane (b) cis-2-pentene (c) trans-2-pentene (d) 1-pentene

33. $A(C_7H_7Cl) \xrightarrow[\text{(ii) Sodalime}/\Delta]{\text{(i) KMnO}_4}$ Chlorobenzene. A forms only one nitro derivative. Hence A is,

(a) Benzyl chloride (b) p-Chlorotoluene
(c) o-Chlorotoluene (d) m-Chlorotoluene

34. Claisen rearrangement of allyl phenyl ether gives a mixture of

(a) o-, m-and p-allyl phenols (b) o-and m-allyl phenols
(c) m-and p-allyl phenols (d) o-and p-allyl phenols.

35. If in diamond, there is a unit cell of carbon atoms as fcc and if carbon atom is sp^3 hybridized, what fractions of void are occupied by carbon atom.

(a) 25% tetrahedral (b) 50% tetrahedral
(c) 25% octahedral (d) 50% octahedral

36. Out of boiling point (I), entropy (II), pH (III) and standard e.m.f. of a cell (IV), intensive properties are-

(a) I, II (b) I, II, III (c) I, III,IV (d) all of these

37. Which metal liberates H_2 with very dilute nitric acid:

(a) Zn (b) Cu (c) Mn (d) Hg

38. Which of the following statements are not correct ?

(a) The number of α and β-particles emitted in neptunium series $\left({}^{237}_{93}Np \rightarrow {}^{209}_{83}Bi\right)$ are 7 and 4, respectively.

(b) The number of α and β-particles emitted in uranium series $\left({}^{238}_{92}U \rightarrow {}^{206}_{83}Pb\right)$ are 8 and 7, respectively.

(c) The number of α and β-particles emitted in actinium series $\left({}^{235}_{92}U \rightarrow {}^{207}_{83}Bi\right)$ are 4 and 7, respectively.

(d) The half-life of a radioactive decay is given as $t_{1/2} = 0.693/\lambda$, where λ is decay constant

39. Reactivity of borazole is greater than that of benzene because:

(a) borazole is non-polar compound (b) borazole is polar compound

(c) borazole has electron in it (d) of localized electrons in it

40. When acetone and chloroform are mixed, hydrogen bonding takes place between them. Such a liquid pair will cause.

(a) No deviations from Raoult's law (b) Negative deviations from Raoult's law

(c) Positive deviations from Raoul'ts law (d) None

41. On heating ammonium dichromate, the gas evolved is

(a) Oxygen (b) Ammonia (c) Nitrous oxide (d) Nitrogen.

42. When ammonia is added to a cupric salt solution, the deep blue colour, so observed is due to the formation of

(a) $[Cu(OH)_4]^{2-}$ (b) $[Cu(NH_3)_4]^{2+}$ (c) $[Cu(OH)_2(NH_3)_2]$ (d) $[Cu(H_2O)_4]^{2+}$

43. Arrange the following in order of their decreasing thermal conductivity

(a) Al, Ag, Cu (b) Cu, Ag, Al (c) Ag, Cu, Al (d) Al, Cu, Ag.

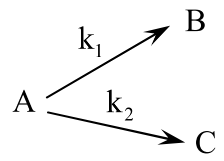
44. In a measurement of quantum efficiency of photo-synthesis in green plants, it was found that 10 quanta of red light of wavelength 6850 Å were needed to release one molecule of O_2. The average energy storage in this process is 112 kcal/mol O_2 evolved. What is the energy conversion efficiency in this experiment? Given 1 cal = 4.18 J, $N_A = 6 \times 10^{23}$, $h = 6.64 \times 10^{-34}$ JS.

(a) 23.5 (b) 26.9 (c) 66.34 (d) 73.1

45. The rate of effusion of two gases 'a' and 'b' under identical condition of temperature and pressure are in the ratio of 2 : 1. What is the ratio of rms velocity of their molecules if T_a and T_b are in the ratio of 2 : 1?

(a) 2 : 1 (b) $\sqrt{2}$: 1 (c) $2\sqrt{2}$: 1 (d) 1 : $\sqrt{2}$

46. For first order parallel reaction k_1 and k_2 are 4 and 2 min^{-1} respectively at 300 K. If the activation energies for the formation of B and C are respectively 30,000 and 33,314 J/mol respectively. The Temperature at which B and C will be obtained in equimolar ratio is:



(a) 757.48 K

(b) 329.77 K

(c) 600 K

(d) None of these

47. One mole of SO_3 was placed in a two litre vessel at a certain temperature. The following equilibrium was established in the vessel.

$$2SO_3\,(g) \rightleftharpoons 2SO_2\,(g) + O_2\,(g)$$

The equilibrium mixture reacted with 0.2 mole $KMnO_4$ in acidic medium. Hence K_c is:

(a) 0.50 (b) 0.25 (c) 0.125 (d) none of these

48. A complex of iron and cyanide ions is 100% ionized at 1 molal. If its elevation in boiling point is 2.08°, then the complex is: (Given K_b = 0.52°C mol^{-1} kg)

(a) $Fe_4[Fe(CN)_6]_3$ (b) $K_3[Fe(CN)_6]$ (c) $K_4[Fe(CN)_6]$ (d) $Fe_3[Fe(CN)_6]_2$

49. Find the standard cell potential involving the cell reaction:

$$In^{2+} + Cu^{+2} \longrightarrow In^{+3} + Cu^{+}, \text{ at } 298\text{ K}$$

Given : $E^\circ_{Cu^{+2}/Cu^{+}} = X_1V$, $E^\circ_{In^{+3}/In^{+}} = X_2V$ $E^\circ_{In^{+2}/In^{+}} = X_3V$

(a) $X_1 + X_3 - X_2$ (b) $(X_1 + X_3 - 2X_2)/3$ (c) $X_1 + X_3 - 2X_2$ (d) $X_1 + X_3 + 2X_2$

50. $SnCl_4$ is a covalent liquid because

(a) e^- clouds of the Cl^- ions are weakly polarized to envelop the cation

(b) electron clouds of the Cl^- ions are strongly polarized to envelop the cation

(c) its molecules are attracted to one another by strong vander Waal's forces

(d) Sn shows inert-pair effect

51. Boron nitride obtained by heating borazole is:

(a) white solid with a diamond like structure

(b) slippery white solid with layered structure similar to that of graphite

(c) covalent liquid and is structurally similar to carbon monoxide

(d) low melting solid with rock-salt like structure molecules

52. If a metal has low oxygen affinity then purification of metal may be carried out by

(a) Liquation (b) Distillation (c) Zone refining (d) Cupellation

53. An original salt solution in acidic medium did not give any precipitate on passing H_2S gas. Such a solution was boiled, reboiled after dilution 3 times. To such a solutions two drops of conc. HNO_3 were added, then heated and water was added. To this resulting solution, NH_4Cl was first added followed by excess of NH_4OH. Finally a green ppt. was obtained. Hence the cation may be:

(a) Al^{+3} (b) Fe^{+2} (c) Fe^{+3} (d) Cr^{+3}

54. Which of the following statement is wrong?

(a) d orbital taking part in dsp^2 is $d_{x^2-y^2}$

(b) d orbital taking part in sp^3d is d_{yz}

(c) d orbitals taking part in sp^3d^2 are $d_{x^2-y^2}$ and d_{z^2}

(d) d orbitals taking part in sp^3d^3 are d_{xy}, d_{z^2} and $d_{x^2-y^2}$

55. $H_3CCOOH \xrightarrow{Br_2/P} (Y) \xrightarrow[(ii)\ H_3O^+]{(i)\ KCN} (X)$

Here (X) is:

(a) Glycollic acid (b) α-hydroxypropionic acid

(c) succinic acid (d) malonic acid

56. In order to distinguish between $C_2H_5NH_2$ and $C_6H_5NH_2$, which of the following reagents is useful

(a) Heinsberg reagent (b) p-napthol

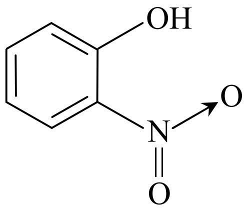
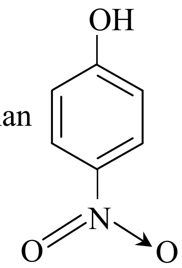
(c) Benzene diazonium chloride (d) None of these

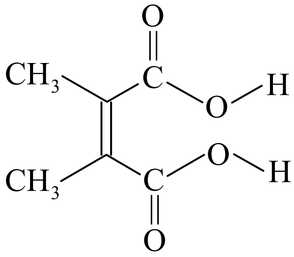
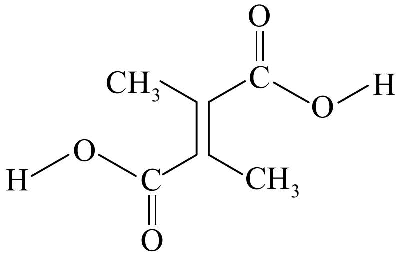
57. The weakest interparticle forces are present in

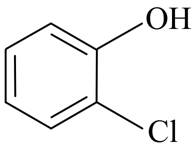
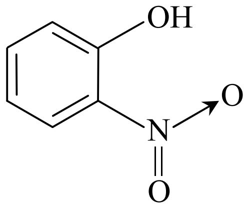
(a) thermosetting polymers (b) thermoplastic polymers

(c) fibres (d) elastomers

58. Which of the following statement is false ?

(a)  has lower boiling point than 

(b)  is less stable than 

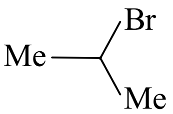
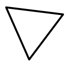
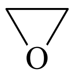
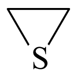
(c)  has greater pKa than 

(d) H_3BO_3 in C_2H_5OH exists in the cage form due to intermolecular hydrogen bonding.

59. Which among the following compounds will not give effervescence with sodium bicarbonate?

(a) C_6H_5COOH (b) $C_6H_5SO_3H$ (c) C_6H_5OH (d) picric acid

60. Which of the following is most reactive for S_N2 reaction?

(a)  (b)  (c)  (d) 

MATHS

SECTION – III

This section contains ***30 questions.*** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which ***ONLY ONE*** is correct. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

61. The line $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-1}{-1}$ intersects the curve $xy=c^2,\ z=0$ if c is equal to :

(a) ± 1 (b) $\pm\frac{1}{3}$ (c) $\pm\sqrt{5}$ (d) none of these

62. If $x^2+y^2+z^2=1,$ then the value of $xy+yz+zx$ lies in the interval :

(a) $\left[\frac{1}{2}, 2\right]$ (b) $[-1, 2]$ (c) $\left[-\frac{1}{2}, 1\right]$ (d) $\left[-1, \frac{1}{2}\right]$

63. For $a\neq b,$ if the equations $x^2+ax+b=0$ and $x^2+bx+a=0$ have a common root, then the value of $(a+b)$ is :

(a) –1 (b) 0 (c) 1 (d) 2

64. If $|z|=1,$ then $\left(\frac{1+z}{1+\bar{z}}\right)$ equals :

(a) z (b) $\bar{z}$ (c) z^{-1} (d) none of these

65. If $S=\tan^{-1}\frac{2.1}{2+1^2+1^4}+\tan^{-1}\frac{2.2}{2+2^2+2^4}+\tan^{-1}\frac{2.3}{2+3^2+3^4}+\ldots$ upto infinite terms, then S equals :

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

66. If $\cos x=\frac{2\cos y-1}{2-\cos y},$ where $x,\ y\in(0,\ \pi),$ then $\tan\frac{x}{2}\cdot\cot\frac{y}{2}$ is equal to :

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{3}}$

67. The most general values of x for which $\sin x + \cos x = \min_{a \in R}\{1, a^2 - 4a + 6\}$ are given by :

(a) $2n\pi, n \in N$
(b) $2n\pi + \frac{\pi}{2}, n \in N$
(c) $n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}, n \in N$
(d) None of the above

68. If x_1, x_2, x_3, x_4 are roots of the equation $x^4 - x^3 \sin 2\beta + x^2 \cos 2\beta - x\cos\beta - \sin\beta = 0$, then $\sum_{i=1}^{4} \tan^{-1} x_i$ is equal to :

(a) β
(b) $\frac{\pi}{2} - \beta$
(c) $\pi - \beta$
(d) $-\beta$

69. A person goes for an examination in which there are four papers with a maximum of m marks from each paper. The number of ways in which one can get $2m$ marks is :

(a) $2m+1$
(b) $\frac{1}{3}(m+1)(2m^2+m+1)$
(c) $\frac{1}{3}(m+1)(2m^2+4m+3)$
(d) None of these

70. The value of $\left\{\frac{3^{2003}}{28}\right\}$, where $\{\cdot\}$ denotes the fractional part, is equal to :

(a) $\frac{15}{28}$
(b) $\frac{5}{28}$
(c) $\frac{19}{28}$
(d) $\frac{9}{28}$

71. Three integers are selected simultaneously from the set of integers $\{1, 2, 3, 4, \ldots, 50\}$. The probability that the selected numbers are consecutive, is equal to :

(a) $\frac{9}{(25)(49)}$
(b) $\frac{6}{(25)(49)}$
(c) $\frac{3}{(25)(49)}$
(d) none of these

72. The equations $x+y+z=6, x+2y+3z=10, x+2y+mz=n$ give infinite number of values of the triplet (x, y, z) if :

(a) $m=3, n \in R$
(b) $m=3, n \neq 10$
(c) $m=3, n=10$
(d) none of these

73. Locus of a point that is equidistant from the lines $x+y-2\sqrt{2}=0$ and $x+y-\sqrt{2}=0$ is :

(a) $x+y-5\sqrt{2}=0$
(b) $x+y-3\sqrt{2}=0$
(c) $2x+2y-3\sqrt{2}=0$
(d) $2x+2y-5\sqrt{2}=0$

74. Length of the chord cut off by $y=2x+1$ from the circle $x^2+y^2=4$, is equal to :

(a) $2\sqrt{\frac{13}{5}}$
(b) $2\sqrt{\frac{17}{5}}$
(c) $2\sqrt{\frac{11}{5}}$
(d) $2\sqrt{\frac{19}{5}}$

75. PQ is any focal chord of the parabola $y^2 = 32x$. The length of PQ can never be less than :

(a) 8 unit (b) 16 unit (c) 32 unit (d) 48 unit

76. The locus of point of intersection of the lines $\sqrt{3}x - y - 4\sqrt{3}t = 0$ and $\sqrt{3}tx + ty - 4\sqrt{3} = 0$ 't' being the parameter, is a hyperbola, whose eccentricity is equal to :

(a) $\sqrt{2}$ (b) 2 (c) $\sqrt{3}$ (d) 3

77. The tangent and normal drawn to the ellipse $x^2 + 4y^2 = 4$ at the point P, meet the x-axis at A and B respectively. If $AB = 2$, then $\cos\theta$ is equal to ('θ' being the eccentric angle of point P) :

(a) $\frac{1}{3}$ (b) $\frac{\sqrt{8}}{3}$ (c) $\frac{2}{3}$ (d) $\frac{\sqrt{5}}{3}$

78. $\lim_{x \to 0} \left\{ \frac{\log_e(1+x)}{x^2} + \frac{x-1}{x} \right\} =$

(a) ∞ (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) none of these

79. If $f(x) = \frac{x}{\sqrt{x+1} - \sqrt{x}}$ be a real valued function then :

(a) $f(x)$ is continuous, but $f'(0)$ does not exist (b) $f(x)$ is differentiable at $x = 0$

(c) $f(x)$ is not continuous at $x = 0$ (d) $f(x)$ is not differentiable at $x = 0$

80. If $5f(x) + 3f\left(\frac{1}{x}\right) = x + 2$ and $y = x\,f(x)$, then $\left(\frac{dy}{dx}\right)_{x=1}$ is equal to :

(a) 14 (b) 7/8 (c) 1 (d) none of these

81. The curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$ touches the straight line $\frac{x}{a} + \frac{y}{b} = 2$ at (a, b) for :

(a) $n = 3$ (b) $n = 2$ (c) any value of n (d) no value of n

82. $\int \frac{dx}{(1+\sqrt{x})\sqrt{x - x^2}}$ is equal to :

(a) $\frac{2(\sqrt{x}-1)}{\sqrt{1-x}} + C$ (b) $\frac{2(1+\sqrt{x})}{\sqrt{1-x}} + C$ (c) $\frac{2(\sqrt{x}-1)}{\sqrt{x-1}} + C$ (d) $\frac{2(1+\sqrt{x})}{\sqrt{x-1}} + C$

83. If $\int_0^1 \frac{e^t}{1+t}dt = a,$ then $\int_0^1 \frac{e^t}{(1+t)^2}dt$ is equal to :

(a) $a-1+\frac{e}{2}$ (b) $a+1-\frac{e}{2}$ (c) $a-1-\frac{e}{2}$ (d) $a+1+\frac{e}{2}$

84. The area of the portion enclosed by the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ and the axes of reference is :

(a) $\frac{a^2}{6}$ (b) a^2 (c) $\frac{a^2}{2}$ (d) $\frac{a^2}{4}$

85. Eccentricity of the hyperbola, satisfying the differential equation $2xy\frac{dy}{dx}=x^2+y^2,$ and passing through the point $(2, 1),$ is equal to :

(a) $2\sqrt{2}$ (b) $\sqrt{2}$ (c) $3\sqrt{2}$ (d) none of these

86. If $\vec{\mathbf{a}}$ is a unit vector such that $\vec{\mathbf{a}}\times(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})=\hat{\mathbf{i}}-\hat{\mathbf{k}},$ then $\vec{\mathbf{a}}$ is equal to :

(a) $-\frac{1}{3}(2\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}})$ (b) $\hat{\mathbf{j}}+\hat{\mathbf{k}}$ (c) $\frac{1}{3}(\hat{\mathbf{i}}+2\hat{\mathbf{j}}+2\hat{\mathbf{k}})$ (d) $\hat{\mathbf{i}}$

87. A variable plane passes through a fixed point (a, b, c) and meets the co-ordinate axes in A, B, C. Locus of the point common to the planes through A, B, C and parallel to co-ordinate plane, is :

(a) $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=1$ (b) $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ (c) $ax+by+cz=1$ (d) none of these

88. If the sum of the hypotenuse and one side of a right angled triangle is 5, then the triangle has maximum area when the angle between the hypotenuse and the side is :

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) none of these

89. In a series of $2n$ observations, half of them equal a and remaining half equal $-a$. If the standard deviation of observations is 2, then $|a|$ equals

(a) $\frac{1}{n}$ (b) $\sqrt{2}$ (c) 2 (d) $\frac{\sqrt{2}}{n}$

90. If p, q, r are simple propositions then $(p\wedge q)\wedge(q\wedge r)$ is true then

(a) p, q, r all false (b) p, q, r are all true

(c) p, q are true r is false (d) p is true and q, r is false

SOLUTION OF AITS JEE (MAIN) FULL TEST – 6

PHYSICS

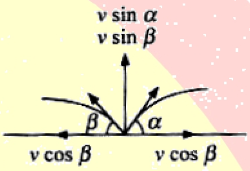
1. **(a)**

Apply $s = \dfrac{\alpha\beta t^2}{2(\alpha+\beta)}$, that is,

$1500 = \dfrac{2\times 3t^2}{2(2+3)}$ or $t^2 = 2500$

$t = 50\text{s}.$

2. **(d)**



$\Delta p_x = m\left[v\cos\alpha - (v\cos\beta)\right]$

$\Delta p_y = m\left[v\sin\alpha - gt - (v\sin\beta - gt)\right]$

$\Delta p = \sqrt{\Delta p_x^2 + \Delta p_y^2} = m\sqrt{(v\cos\alpha + v\cos\beta)^2 + (v\sin\alpha + v\sin\beta)^2}$

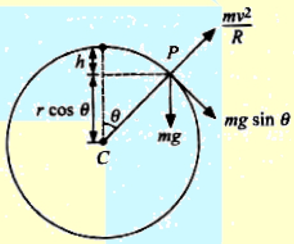
$= mv\sqrt{2 + 2[\cos\alpha\cos\beta - \sin\alpha\sin\beta]} = mv\sqrt{2[1+\cos(\alpha+\beta)]} = 2mv\cos\dfrac{\alpha+\beta}{2}$

3. **(b)**

Assume the particle leaves from the point where it has traveled down a distance h

$h = r - r\cos\theta.$

velocity at point P is given by



$\dfrac{mv^2}{2} = mgh = mg(r - r\cos\theta)$

or $\dfrac{mv^2}{r} = 2mg(1-\cos\theta)$

If the particle leaves at P then $mg\cos\theta = \dfrac{mv^2}{r}$

or $mg\cos\theta = 2mg(1-\cos\theta)$ or $3\cos\theta = 2$ or $\cos\theta = \dfrac{2}{3}$

$h = r - r\cos\theta = r - \dfrac{2r}{3} = r/3.$

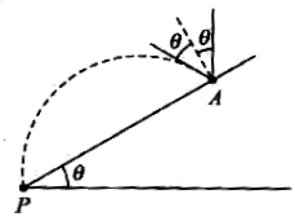
4. **(b)**

$v = \sqrt{2gh}$ … (1)

Applying conservation of momentum the ball will recoil with velocity v making an angle θ with the perpendicular to the plane.

Along horizontal direction

$AP = v\sin\theta\, t + \dfrac{g}{2}\sin\theta\, t^2$ … (2)



Along vertical direction displacement is zero

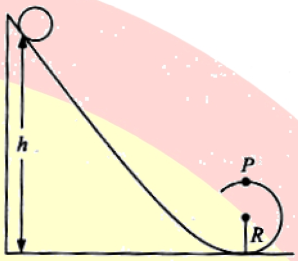
$0 = v\cos\theta\, t - \dfrac{1}{2}g\cos\theta\, t^2$ or $t = \dfrac{2v}{g}$ … (3)

Substituting the value of t from (3) in eq. (2)

$$AP = v\sin\theta\left(\frac{2v}{g}\right)+\frac{g}{2}\sin\theta\left(\frac{2v}{g}\right)^2 = \frac{4v^2}{g}\sin\theta = 8h\sin\theta \text{ along the incline.}$$

5. **(d)**

The minimum velocity at P, top of the loop should be $v=\sqrt{g(R-r)}=r\omega$



$$mgh = \frac{1}{2}mv^2+\frac{1}{2}I\omega^2+mg(2R-r)$$

$$=\frac{1}{2}mg(R-r)+\frac{1}{2}\left(\frac{2}{5}\right)mr^2\omega^2+mg(2R-r)$$

$$=\frac{7}{10}mg(R-r)+mg(2R-r)=\frac{mg}{10}(27R-17r)$$

or $h=\frac{1}{10}(27R-17r)$.

6. **(b)**

$$\Delta E = 4\pi R^2 T\left[n^{1/3}-1\right]$$

$$=4\times 3.14\times 10^{-4}\times 35\times 10^{-3}[99] = 4.356\times 10^{-3}\,\text{J}$$

7. **(c)**

$A_1v_1 = A_2v_2$ or $v_2 = \frac{A_1}{A_2}v_1$

$v_2^2 = v_1^2 + 2gh$ and $A_2 = A_1\frac{v_1}{v_2}$

or $A_2 = \frac{A_1v_1}{\sqrt{v_1^2+2gh}} = \frac{10^{-4}\times 1}{\sqrt{1+2\times 10\times .15}}$

$= 5\times 10^{-5}\,\text{m}^2$.

8. **(c)**

Total clearance = 0.05 mm × 2 = 0.1 mm

$d(\alpha_2-\alpha_1)\Delta T = 0.1\text{mm} = 0.01\text{cm}$

$10\left(0.4\times 10^{-5}\right)\Delta T = 0.01$ or $\Delta T = 250$

or $T = 250+20 = 270°\text{C}$

9. **(b)**

10. **(b)**

$$C_{V\text{mix}} = \frac{n_1C_{V1}+n_2C_{V2}}{n_1+n_2} = \frac{\frac{5}{2}R\times 3+\frac{3}{2}R\times 1}{4} = \frac{18R}{8} = \frac{9R}{4}$$

$$C_{P\text{mix}} = C_{V\text{mix}} + R = \frac{13R}{4}$$

11. **(c)**

$\tau = kyl = kl^2\theta \quad \tau = \frac{Ml^2}{3}\alpha$ therefore, $\frac{Ml^2}{3}\alpha = -kl^2\theta$ or $\alpha = \frac{-3kl^2}{Ml^2}\theta \quad \omega^2 = \frac{3k}{M}$

$T = 2\pi\sqrt{\frac{M}{3k}}$.

12. **(c)**

$$M_{\text{mix}} = \frac{n_1M_1 + n_2M_2}{n_1+n_2} = \frac{1\times4+2\times32}{1+2} = \frac{68}{3}$$

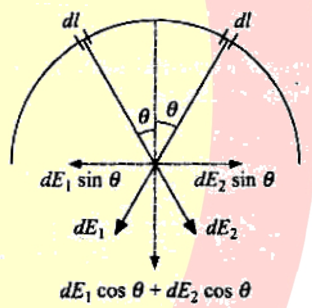
$$C_{V(\text{mixture})} = \frac{n_1C_{V1}+n_2C_{V2}}{n_1+n_2} = \frac{\left(1\times\frac{3}{2}+2\times\frac{5}{2}\right)R}{1+2} = \frac{13}{6}R$$

$$C_{P(\text{mix})} = C_V + R = \frac{19}{6}R;\ \frac{C_P}{C_V} = \frac{19}{13}R$$

$$v = \sqrt{\frac{19}{13}\times\frac{8.31\times300}{\frac{68}{3}\times10^{-3}}} = 400.9\,\text{ms}^{-1}.$$

13. **(d)**

Consider two small elements of length dl each. Charge dq on each element



$$dq = \frac{Q}{l}dl = \frac{Q(rd\theta)}{\pi r} = \frac{Qd\theta}{\pi}$$

$\because\ |dE_1| = |dE_2|$, resolve dE_1 and dE_2 their sin components cancel out.

$$dE = 2dE_1\cos\theta = \frac{2Qd\theta}{\pi4\pi\varepsilon_0r^2}\cos\theta$$

and $E = \frac{2Q}{4\pi^2\varepsilon_0r^2}\int_0^{\pi/2}\cos\theta d\theta$ or $E = \frac{2Q}{4\pi^2\varepsilon_0\left(\frac{l}{\pi}\right)^2}[\sin90-\sin0] = \frac{Q}{2\varepsilon_0l^2}$

14. **(d)**

15. **(c)**

$Q = Q_0e^{-t/RC}$ and $\frac{dQ}{dt} = i = \frac{Q_0}{R_C}e^{-t/RC} = \frac{20\times20\times10^{-6}}{10\times20\times10^{-6}}e^{-5}1 = 2(0.0067) = 13.4\text{mA}$

16. **(c)**

Assume a hypothetical cylinder of radius x and thickness dx then

$$\int dR = \int_a^b \rho\frac{dx}{2\pi xl}$$

$R = \frac{\rho\log_e\frac{b}{a}}{2\pi l}$ and $I = \frac{\varepsilon}{R} = \frac{\varepsilon}{\rho}\frac{2\pi l}{\log_e\frac{b}{a}}$

17. **(a)**

$$I = \frac{6}{R_{11}+1} = \frac{6}{2+1} = 2\text{A}$$

$$R_{11} = \frac{6\times 3}{6+3} = 2\pi$$

Current through 3Ω resistor $I' = (I\times 6)/9 = 4/3\text{A}$

$$(mC)\Delta T = I'^2 Rt$$

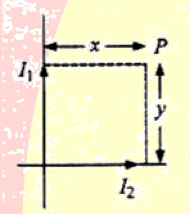
$$2000\times\Delta T = \left(\frac{4}{3}\right)^2 \times 3\times 15\times 60$$

$$\Delta T = 2.4°\text{C}.$$

18. **(a)**

Magnetic field could the zero in 1st or 3rd quadrant.

$$\frac{\mu_0 I_1}{2\pi x} = \frac{\mu_0 I_2}{2\pi y} \text{ or } I_1 = \frac{x}{y} I_2.$$



19. **(b)**

diamagnetic materials show a feeble repulsion.

20. **(b)**

21. **(d)**

$$I_{\text{rms}} = \sqrt{6^2+8^2} = 10\text{A}$$

$$V_R = I_{\text{rms}} \times R = 10\times 40 = 400\text{V}$$

22. **(c)**

$$\frac{H}{2} + \frac{H}{\omega_{\mu_a}} = \frac{H}{2} + \frac{H}{1/\mu}$$

23. **(c)**

24. **(a)**

$$hf = \frac{6.625\times 10^{-34}\times 1.4\times 10^{15}}{1.6\times 10^{-19}} = 5.8\text{eV};$$

$$(kE)_{\max} = hf - \phi$$

$$3 = 5.8 - 2.8.$$

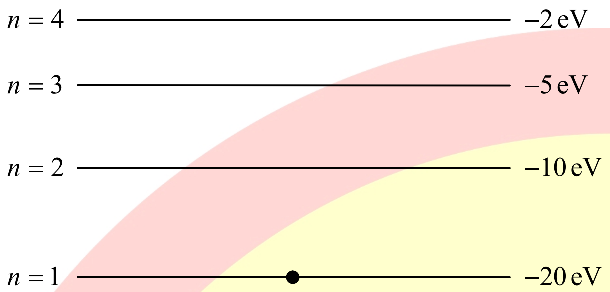
Since anode voltage is 3V, the electrons emitted with zero KE will acquire an energy $= 3\text{eV}$ and the electrons emitted with 3eV will acquire $3+3 = 6\text{eV}$

$\therefore$ min $KE = 3\text{eV}$ and max $KE = 6\text{eV}$.

25. **(d)**

$$E(ev) = \frac{1240}{860} = 1.44\text{eV}$$

$$E_{\text{net}} = -6.52 + 1.44 = -5.08\text{eV}$$



26. **(d)**

$$Q = [7.016004 + 1.007825 - 2(4.002603)] \times 931.5 \;\; = 0.018623 \times 931.5 = 17.35\,\text{MeV}$$

27. **(b)**

$$A_V = \frac{R_f}{R_i} = \frac{100\,k\Omega}{1\,k\Omega} = 100$$

28. **(a)**

$$\delta = m_f \times f_m = 50 \times 500 = 25 \text{ kHz}$$

$$kf_C = \frac{\delta}{E_m} = \frac{25}{2.5} = 10\,\text{kHz/V}$$

case (ii) $\delta_{\text{new}} = kf_C(E_{m2}) = 10 \times 4.5 = 45\,\text{kHz}$ and $m_f = \frac{45 \times 10^3}{200} = 225.$

29. **(a)**

30. **(b)**

CHEMISTRY

31. **(a)** 32. **(c)** 33. **(b)** 34. **(d)**

35. **(b)** 36. **(c)** 37. **(c)** 38. **(c)**

39. **(b)** 40. **(b)** 41. **(d)** 42. **(b)**

43. **(c)**

44. **(b)**

$$E = \frac{hc}{\lambda} = 2.9 \times 10^{-19} \text{ J}$$

Total energy of 10 quanta = $10 \times 2.9 \times 10^{-19}$J

Energy stored for process $= \frac{112 \times 4.18 \times 10^3}{6 \times 10^{23}} = 7.8 \times 10^{-19}$ J

% efficiency $= \frac{7.8 \times 10^{-19}}{29 \times 10^{-19}} \times 100 = 26.9\%$

45. **(c)**

$$\frac{r_a}{r_b}=\frac{2}{1}=\sqrt{\frac{M_b}{M_a}}$$

$$V_{rms}\propto\sqrt{\frac{T}{M}}\qquad \left(\text{As } V_{rms}=\sqrt{\frac{3RT}{M}}\right)$$

$$\frac{V_{rms(a)}}{V_{rms(b)}}=\sqrt{\frac{T_a\times M_b}{T_b\times M_a}}=\frac{2}{1}\times\frac{\sqrt{2}}{1}=\frac{2\sqrt{2}}{1}$$

46. **(b)**

$$\ell n\frac{k_1'}{k_1}=\frac{E_1}{R}\left[\frac{1}{T_1}-\frac{1}{T_2}\right]\qquad \ldots(i)$$

$$\ell n\frac{k_2'}{k_2}=\frac{E_2}{R}\left[\frac{1}{T_1}-\frac{1}{T_2}\right]\qquad \ldots(ii)$$

Eqn. (ii) – Eqn. (i)

$$\ell n\frac{k_2'\times k_1}{k_2\times k_1'}=\frac{(E_2-E_1)}{R}\left[\frac{1}{T_1}-\frac{1}{T_2}\right]$$

(For equimolar formation of B and C, $k_2'=k_1'$)

$$\ell n\left(\frac{k_1}{k_2}\right)=\left(\frac{8314}{8.314}\right)\left(\frac{T_2-300}{300\times T_2}\right)$$

$$\ell n2=\left(\frac{8314}{8.314}\right)\left(\frac{T_2-300}{300\times T_2}\right)$$

$$T_2=329.77\,K$$

47. **(c)**

$$2SO_3(g)\rightleftharpoons 2SO_2(g)+O_2(g)$$

at equilibrium (1 – 2x) (2x) (x)

Only SO_2 will oxidized.

Equivalent of SO_2 = Equivalent of $KMnO_4$

2x × 2 = 0.2 × 5

2x = 0.5

$$K_c=\frac{\left[\frac{0.5}{2}\right]^2\left[\frac{0.25}{2}\right]}{\left[\frac{0.5}{2}\right]^2}=0.125$$

48. **(b)**

$\Delta T_b = i \times k_b \times m$

$2.08 = 0.52 \times 1 \times i$

$i = 4$

It means salt on dissociation gives 4 ions. Thus the salt that gives 4 ions is $K_3[Fe(CN)_6]$.

49. **(c)**

$Cu^{+2} + e^- \longrightarrow Cu^+;$ $\quad E^\circ = X_1 V \quad \Delta G_1^\circ = -FX_1 \quad \ldots(i)$

$In^{+2} + 2e^- \longrightarrow In^{+1};$ $\quad E^\circ = X_2 V \quad \Delta G_2^\circ = -2FX_2 \quad \ldots(ii)$

$In^{+2} + e^- \longrightarrow In^+;$ $\quad E^\circ = X_3 V \quad \Delta G_3^\circ = -FX_3 \quad \ldots(iii)$

From Eqn. (i) + (iii) – (ii)

$In^{+2} + Cu^{2+} \longrightarrow In^{+3} + Cu^+ \qquad \Delta G^\circ = -FE^\circ$

$\Delta G^\circ = -F(X_1 + X_3 - 2X_2) = -FE^\circ$

$E^\circ = (X_1 + X_3 - 2X_2)$

50. **(b)**

Due to Fajan's rule.

51. **(b)**

52. **(b)**

53. **(d)**

Green ppt. is – $Cr(OH)_3$

54. **(b)**

55. **(d)**

$$H_3C-\overset{\overset{\large O}{\|}}{C}-OH \xrightarrow[Br_2/P]{HVZ} \underset{(Y)}{BrCH_2-\overset{\overset{\large O}{\|}}{C}-OH} \xrightarrow[H_3O^+]{KCN} \underset{(X)\ \text{Malonic acid}}{OH-\overset{\overset{\large O}{\|}}{C}-CH_2-COOH}$$

56. **(b)**

1° aromatic amine on diazotisation followed by coupling with β-napthol gives azo dye test.

57. **(d)**

58. **(d)**

(a) Viscosity α intermolecular attraction

α intermolecular H-bonding tendency

(b) No intramoecular H-bonding in the cis-isomer due to loss in planarity as a result of steric and electronic repulsion

$\therefore$ stability α symmetry.

(c) Even though o-nitrophenol shows intramolecular H-bonding but has greater – R and – I effect than –Cl.

(d) $H_3BO_3 + 3\ C_2H_5OH \rightarrow B(OC_2H_5)_3 + 3H_2O$.

$\therefore$ No cage lattice structure of H_3BO_3.

59. **(c)**

Because phenol is less acidic than H_2CO_3.

60. **(d)**

S is less electronegative and therefore more reactive towards S_N2

MATHS

61. **(c)**

We have, $z = 0$ for the point where the line intersects the curve

$$\therefore \frac{x-2}{3} = \frac{y+1}{2} = \frac{0-1}{-1} \Rightarrow \frac{x-2}{3} = 1 \text{ and } \frac{y+1}{2} = 1 \Rightarrow x = 5 \text{ and } y = 1$$

Put these values in $xy = c^2$

we get, $5 = c^2 \Rightarrow c = \pm\sqrt{5}$

62. **(c)**

Let $xy + yz + zx = \lambda$. Then, $x^2 + y^2 + z^2 - \lambda$

$$= \frac{1}{2}\left[(x-y)^2 + (y-z)^2 + (z-x)^2\right] \geq 0 \Rightarrow 1 - \lambda \geq 0$$

Again, $(x+y+z)^2 = x^2 + y^2 + z^2 + 2\lambda = 1 + 2\lambda \Rightarrow 1 + 2\lambda \geq 0$

63. **(a)**

64. **(a)**

$$\frac{1+z}{1+\bar{z}} = \frac{z(1+z)}{z+z\bar{z}} = \frac{z(1+z)}{z+|z|^2} = \frac{z(1+z)}{(z+1)} = z \quad (\because |z| = 1)$$

65. **(d)**

66. **(b)**

$$\cos x = \frac{2\cos y - 1}{2 - \cos y} \Rightarrow \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \frac{\dfrac{2\left(1-\tan^2\frac{y}{2}\right)}{1+\tan^2\frac{y}{2}} - 1}{2 - \dfrac{1-\tan^2\frac{y}{2}}{1+\tan^2\frac{y}{2}}}$$

$$\Rightarrow 6\tan^2\frac{y}{2} = 2\tan^2\frac{x}{2} \Rightarrow \tan\frac{x}{2} \cdot \cot\frac{y}{2} = \sqrt{3} \quad [\text{as } x,\ y \in (0,\ \pi)]$$

67. **(c)**

$\sin x+\cos x=\min\limits_{a\in R}\left\{1,\ a^2-4a+6\right\}$

$\because\ a^2-4a+6=(a-2)^2+2>2$

For all a

$\therefore\ \sin x+\cos x=1$

$\sin\left(x+\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}\quad\Rightarrow x+\frac{\pi}{4}=n\pi+(-1)^n\frac{\pi}{4}\quad\Rightarrow x=n\pi+(-1)^n\frac{\pi}{4}-\frac{\pi}{4}$

68. **(b)**

69. **(c)**

Required number

= coefficient of x^{2m} in $\left(x^0+x^1+...+x^m\right)^4$

= coefficient of x^{2m} in $\left(\frac{1-x^{m+1}}{1-x}\right)$ = coefficient of x^{2m} in $\left(1-x^{m+1}\right)^4(1-x)^{-4}$

= coefficient of x^{2m} in $\left(1-4x^{m+1}+6x^{2m+2}+...\right)(1-x)^{-4}={}^{2m+3}C_{2m}-4\cdot{}^{m+2}C_{m-1}$

$=\frac{(2m+1)(2m+2)(2m+3)}{6}-\frac{4m(m+1)(m+2)}{6}=\frac{(m+1)\left(2m^2+4m+3\right)}{3}$

70. **(c)**

$3^{2003}=3^{2001}\cdot 3^2=9(27)^{667}=9(28-1)^{667}$

$=9\left({}^{667}C_0 28^{667}+{}^{667}C_1(28)^{666}+...+{}^{667}C_{667}(-1)^{667}\right)$ that means if we divide 3^{2003} by 28, remainder is 19.

Thus, $\left\{\frac{3^{2003}}{28}\right\}=\frac{19}{28}$

71. **(c)**

Possible set of three consecutive numbers are $\{1, 2, 3\}, \{2, 3, 4\}, \{3, 4, 5\}, ..., \{48, 49, 50\}$. Which are 48 in number.

Thus, required probability $=\frac{{}^{48}C_1}{{}^{50}C_3}=\frac{3}{(25)(49)}$

72. **(c)**

73. **(c)**

For any point $P(x, y)$ that is equidistant from given line, we have

$x+y-\sqrt{2}=-\left(x+y-2\sqrt{2}\right)\ \Rightarrow\ 2x+2y-3\sqrt{2}=0$

74. **(d)**

75. **(c)**

Length of focal chord is $a\left(t+\frac{1}{t}\right)^2$, if $\left(at^2, 2at\right)$ is one extremity of the parabola $y^2=4ax$

$\therefore\ t+\frac{1}{t}\ge 2(A.M.\ge G.M.)\ \Rightarrow a\left(t+\frac{1}{t}\right)^2\ge 4a$

Here, $4a=32$

76. **(b)**

$x\sqrt{3}-y-4\sqrt{3}t=0\ \Rightarrow t=\frac{1}{4\sqrt{3}}\left(x\sqrt{3}-y\right)$

Also, $\sqrt{3}tx+ty-4\sqrt{3}=0\ \Rightarrow t=\frac{4\sqrt{3}}{\left(x\sqrt{3}+y\right)}\ \Rightarrow 48=3x^2-y^2\ \Rightarrow \frac{x^2}{16}-\frac{y^2}{48}=1$

If 'e' be it's eccentricity, then $e^2=1+\frac{48}{16}=4\ \Rightarrow e=2$

77. **(c)**
78. **(b)**

Required limit

$$=\frac{\log_e(1+x)+x^2-x}{x^2}=\lim_{x\to 0}\frac{\left(x-\frac{x^2}{2}+\frac{x^3}{3}-...\right)+x^2-x}{x^2}=\lim_{x\to 0}\frac{\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+...}{x^2}=\lim_{x\to 0}\frac{x^2\left(\frac{1}{2}+\frac{x}{3}-...\right)}{x^2}=\frac{1}{2}$$

79. **(b)**

$f(x)=\frac{x\left(\sqrt{x^2+1}+\sqrt{x}\right)}{x+1-x}=x\left(\sqrt{x^2+1}+\sqrt{x}\right)$. As $f(x)$ is real valued, $x\ge 0$

$\therefore f(x)$ is differentiable at $x=0$ if

$\lim_{h\to 0}\frac{(0+h)-f(0)}{h}$ is finite and definite $=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\frac{h\left(\sqrt{h^2+1}+\sqrt{h}\right)}{h}=1$

A function which is finitely differentiable is also continuous.

80. **(b)**
81. **(c)**
82. **(a)**

$$I=\int\frac{dx}{\left(1+\sqrt{x}\right)\sqrt{x-x^2}}=\int\frac{\left(1-\sqrt{x}\right)}{(1-x)\sqrt{x}(1-x)^{1/2}}dx=\int\frac{\left(1-\sqrt{x}\right)dx}{\sqrt{x}(1-x)^{3/2}}$$

$$=\int\frac{dx}{x^2\left(\frac{1}{x}-1\right)^{3/2}}-\int\frac{dx}{(1-x)^{3/2}}=-\int\frac{d\left(\frac{1}{x}-1\right)}{\left(\frac{1}{x}-1\right)^{3/2}}-\int\frac{dx}{(1-x)^{3/2}}$$

$$=\frac{2}{\left(\frac{1}{x}-1\right)^{3/2}}-\frac{2}{(1-x)^{3/2}}+C=\frac{2}{\sqrt{1-x}}+\frac{2\sqrt{x}}{\sqrt{1-x}}+C=\frac{2\left(\sqrt{x}-1\right)}{\sqrt{1-x}}+C$$

83. **(b)**

84. **(a)**

85. **(b)**

$$2xy\frac{dy}{dx} = x^2 + y^2$$

$$\Rightarrow 2xy\,dy - y^2dx = x^2dx \;\Rightarrow \frac{x\cdot 2y\,dy - y^2dx}{x^2} = dx \;\Rightarrow d\left(\frac{y^2}{x}\right) = dx \Rightarrow \frac{y^2}{x} = x + c$$

Since it passes through (2, 1), therefore

$$\frac{1}{2} = 2 + c \;\Rightarrow c = -\frac{3}{2}$$

Thus, hyperbola is,

$$y^2 = x^2 - \frac{3}{2}x$$

i.e., $\left(x - \frac{3}{4}\right)^2 - y^2 = \frac{9}{16}$

This hyperbola is clearly rectangular in nature. Hence, it's eccentricity is $\sqrt{2}$.

86. **(a)**

87. **(a)**

Let the variable plane be $\frac{x}{a_1} + \frac{y}{b_1} + \frac{z}{c_1} = 1$

Since, it passes through (a, b, c), therefore

$$\frac{a}{a_1} + \frac{b}{b_1} + \frac{c}{c_1} = 1$$

$$A \equiv (a_1, 0, 0),\ B \equiv (0, b_1, 0),\ C \equiv (0, 0, c_1)$$

Let $P \equiv (x, y, z)$ be the point common to the planes passing through A, B, C are parallel to co-ordinate planes, then

$$x = a_1,\ y = b_1,\ z = c_1$$

Locus of 'P' is

$$\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 1$$

88. **(b)**

89. **(c)**

90. **(b)**